ENTRANCE EXAMINATION FOR ADMISSION, MAY 2012.

M.Sc. (Mathematics)

COURSE CODE : 372

Register Number :

Signature of the Invigilator (with date)

COURSE CODE : 372

Time : 2 Hours | Max : 400 Marks

Instructions to Candidates :

1. Write your Register Number within the box provided on the top of this page and fill in the page 1 of the answer sheet using pen.
2. Do not write your name anywhere in this booklet or answer sheet. Violation of this entails disqualification.
3. Read each of the question carefully and shade the relevant answer (A) or (B) or (C) or (D) in the relevant box of the ANSWER SHEET using HB pencil.
4. Avoid blind guessing. A wrong answer will fetch you –1 mark and the correct answer will fetch 4 marks.
5. Do not write anything in the question paper. Use the white sheets attached at the end for rough works.
6. Do not open the question paper until the start signal is given.
7. Do not attempt to answer after stop signal is given. Any such attempt will disqualify your candidature.
8. On stop signal, keep the question paper and the answer sheet on your table and wait for the invigilator to collect them.
9. Use of Calculators, Tables, etc. are prohibited.

Notation: **R** – Real line, **Q**- Set of rationals, **N**- Set of natural numbers and **C**- Set of Complex numbers, **Z**- Set of integers, ϕ- empty set.

For a set E, $\overline{E}$ - closure of E, E^C - complement of E and sp(E)- span of E .

If A is a matrix, adj A, det(A) and A^T denote the adjoint, determinant and the transpose of the matrix A respectively.

$i=\sqrt{-1}$ and Re z and Im z denote the real and imaginary part of a complex number z.

Instructions to candidates :

(i) Answer all questions.

(ii) Each correct answer carries 4 marks and each wrong answer carries -1 mark.

(iii) IMPORTANT: Mark the correct statement, unless otherwise specified.

1. If $\cos\alpha, \cos\beta, \cos\gamma$ are the direction cosines of a straight line, then $\sin^2\alpha + \sin^2\beta + \sin^2\gamma =$

(A) 1 (B) 0 (C) 2 (D) $\frac{1}{2}$

2. The equation of the plane through the point (1,1,1) and the straight line given by $x+2y-z+1=0=3x-y+4z+3$ is

(A) $y-z=0$ (B) $x-y=0$ (C) $x-z=0$ (D) $x+y=0$

3. Two lines $\frac{x-x_1}{l_1}=\frac{y-y_1}{m_1}=\frac{z-z_1}{n_1}, \frac{x-x_2}{l_2}=\frac{y-y_2}{m_2}=\frac{z-z_2}{n_2}$ are coplanar if

(A) $l_1l_2+m_1m_2+n_1n_2=0$

(B) $\begin{vmatrix} x_1-x_2 & y_1-y_2 & z_1-z_2 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix}=0$

(C) $\begin{vmatrix} x_1-x_2 & y_1-y_2 & z_1-z_2 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix}\neq 0$

(D) $\begin{vmatrix} x_1-x_2 & y_1-y_2 & z_1-z_2 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix}=1$

4. The radius of the sphere $ax^2+ay^2+az^2+2ux+2vy+2wz+d=0$ is

(A) $\sqrt{u^2+v^2+w^2-d}$

(B) $\sqrt{\frac{u^2}{a^2}+\frac{v^2}{a^2}+\frac{w^2}{a^2}-\frac{d}{a}}$

(C) d

(D) u

5. $\cosh 2\theta =$

(A) $\cosh^2 x - \sinh^2 x$

(B) $\sinh^2 x - \cosh^2 x$

(C) $\cosh^2 x + \sinh^2 x$

(D) $2\sinh x \cosh x$

6. $\lim_{\theta \to 0} \frac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1} =$

(A) 1 (B) 0 (C) 2 (D) 1/2

7. The direction cosines of a line parallel to the z-axis are

(A) 0,1,0 (B) 0,0,1 (C) 0,0,0 (D) 1,0,0

8. Among those below, the triple that cannot form the direction cosines of any straight line is

(A) (0,1,0)

(B) (0,0,1)

(C) $(0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$

(D) $(\frac{1}{2}, 0, \frac{1}{2})$

9. The number of ways in which six children can be seated in a toy car having 5 seats , excluding the driver's seat, if one of the three girls must steer is

(A) $3 \times 8!$ (B) $6 \times 5!$ (C) $3 \times 5!$ (D) $6!$

10. How many committees of 5 people can be chosen from 20 men and 12 women if at least four women must be on each committee?

(A) 10,692 (B) 10, 690 (C) 1069 (D) 1,06,900

11. The coefficient of x^{16} in the expansion of $\left(2x^2 - \frac{x}{2}\right)^{12}$ is

(A) $\frac{490}{16}$ (B) $\frac{491}{16}$ (C) $\frac{494}{16}$ (D) $\frac{495}{16}$

12. For any natural number n, $\binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \ldots + \binom{n}{n} =$

(A) 3^n (B) $3n$ (C) 2^n (D) $2n$

13. The number of ways to put r identical marbles into n number of boxes (a box can contain any number of marbles) is

(A) $\binom{n}{r}$ (B) $P(n,r)$ (C) $\binom{n+r-1}{r}$ (D) n

14. The solution of the equation $\frac{d^3y}{dx^3} - 3\frac{d^2y}{dx^2} + 3\frac{dy}{dx} - y = 0$ is

(A) $e^x\left(c_1x^2 + c_2x + c_3\right)$

(B) $e^x\left(c_1x^2 + c_2\right)$

(C) $e^x\left(c_1x^2 + x(c_2 + c_3)\right)$

(D) $e^{2x}\left(c_1x^2 + c_2x + c_3\right)$

15. The solution of the equation $\frac{d^5y}{dx^5} + \frac{d^3y}{dx^3} = 0$ is

(A) $c_1e^{2x} + c_2e^x + \left(c_3 + c_4x + c_5x^2\right)e^{0.x}$ (B) $c_1e^{-x} + c_2e^x$

(C) $c_1e^{-x} + c_2e^x + \left(c_3 + c_4x + c_5x^2\right)e^{3.x}$ (D) $c_1e^{-x} + c_2e^x + \left(c_3 + c_4x + c_5x^2\right)e^{0.x}$

16. $\int_{-a}^{a} x^3\sqrt{a^2 - x^2}\,dx =$

(A) a^3 (B) $2a^3$ (C) 0 (D) $2a^3$

17. If $f(2a - x) = -f(x)$, then $\int_{0}^{2a} f(x)dx =$

(A) $2a$ (B) a (C) 0 (D) a^2

18. $\int \frac{x^{n-1}}{1 + x^n}dx =$

(A) $-\frac{1}{n}\log\left(1 + x^n\right)$ (B) $\frac{1}{n}\log\left(1 + x^n\right)$

(C) $\log\left(1 + x^n\right)$ (D) $\frac{1}{n-1}\log\left(1 + x^n\right)$

19. If L denotes the Laplace transform, then $L^{-1}\left[\frac{1}{(s+3)^4}\right] =$

(A) e^{-3t} (B) $t^3 e^{-3t}$ (C) $\frac{t^3}{3!}e^{-3t}$ (D) $\frac{t^3}{3!}e^{3t}$

20. Let X be a vector space, V_1 and V_2 be subspaces of X. Then

(A) $\dim(V_1+V_2)=\dim V_1+\dim V_2$ (B) $\dim(V_1-V_2)=\dim(V_1+V_2)$

(C) $\dim(V_1-V_2)=\dim V_1-\dim V_2$ (D) $\dim(V_1\cap V_2)=\dim(V_1-V_2)$

21. A sequence $(x_n)_{n=1}^{\infty}$ of reals does not have a convergent subsequence if

(A) $\lim_{n\to\infty} x_n = \infty$ (B) the sequence is unbounded

(C) $\liminf_n x_n < \limsup_n x_n$ (D) it is not convergent

22. (A) If a sequence of reals is not monotone then everyone if its subsequence is not monotone

(B) If a sequence of reals is unbounded then everyone of its subsequence is unbounded

(C) If a sequence of reals is bounded then everyone of its subsequence is bounded

(D) If a sequence of reals is not convergent everyone of its subsequence is not convergent

23. If $a_n = (n-10^{100})+1/n$, n≥1, then

(A) $\liminf_n a_n = -\infty$ (B) $\limsup_n a_n = \infty$

(C) $\limsup_n a_n = 0$ (D) $\liminf_n a_n = 0$

24. If $(x_n)_{n=1}^{\infty}$ is a sequence of reals such that $|x_n - x_{n+1}| < 1/n$ for all positive integers n then the sequence $(x_n)_{n=1}^{\infty}$

(A) is convergent (B) is Cauchy but not convergent

(C) is bounded but not convergent (D) need not be bounded

25. The series $\sum_{n=1}^{\infty}(-1)^n n^3 e^{-n}$

(A) is absolutely convergent

(B) is convergent but not absolutely convergent

(C) is not convergent

(D) diverges to ∞

26. Let $f(x,y)=\begin{cases} x+y \text{ } if \text{ } x=0 \text{ } or \text{ } y=0 \\ 1 \quad otherwise \end{cases}$

(A) $\frac{\partial f}{\partial x}(0,0)=0$ *and* $\frac{\partial f}{\partial y}(0,0)=0$

(B) $\frac{\partial f}{\partial x}(0,0)=0$ *and* $\frac{\partial f}{\partial y}(0,0)=1$

(C) $\frac{\partial f}{\partial x}(0,0)=1$ *and* $\frac{\partial f}{\partial y}(0,0)=0$

(D) $\frac{\partial f}{\partial x}(0,0)=1=\frac{\partial f}{\partial y}(0,0)$

27. If $f(x)=\frac{1}{2-e^{\frac{1}{x}}}$ $,x\in \mathbf{R}$, Then

(A) $\lim_{x\to\infty} f(x)=-1$

(B) $\lim_{x\to\infty} f(x)=1$

(C) $\lim_{x\to\infty} f(x)=2$

(D) $\lim_{x\to\infty} f(x)=0$

28. If $f(x)=\{ \begin{matrix} x\sin\frac{1}{x}+\frac{Sinx}{x} & for \text{ } x\neq 0 \\ a & x=0 \end{matrix}$ then f is continuous at 0

(A) for no real value of a

(B) if a = 0

(C) if a = 1

(D) if a = –1

29. If f is a surjective map from **R** onto the set of rationals, then

(A) f is neither continuous nor injective

(B) f can be continuous but not injective

(C) f can be injective but not continuous

(D) f can be both continuous and injective

30. $(\cos x)(\cos 2x)(\cos 2^2 x)\text{.........}\left(\cos 2^n x\right)$ equals

(A) $\frac{\sin 2^n x}{\sin x}$

(B) $\frac{\sin 2^{n+1} x}{2^{n+1}\sin x}$

(C) $\frac{\cos 2^n x}{2^n}$

(D) $\cos 2^{n+1} x$

31. The series $\sum_{n=1}^{\infty} \frac{(-1)^n}{\left|\sqrt{n}\right|}$

(A) is not convergent

(B) is convergent but not absolutely convergent

(C) is absolutely convergent

(D) diverges to ∞

32. A basis for the subspace of $\mathbf{R}^4$ spanned by the vectors (1,2,-1,0), (4,8,-4,-3), and (6,12,-6,-3) is

(A) $\{(1,2,-1,0), (-3,-6,3,0)\}$ (B) $\{(-1,-2,1,\frac{3}{2}), (-2,-4,2,\frac{3}{2})\}$

(C) $\{(-1,-2,1,\frac{3}{2}), (-2,-4,2,3)\}$ (D) $\{(1,0,0,0), (0,0,1,0)\}$

33. Mark the **wrong** statement.

(A) det(AB) = det(A) det(B)

(B) $\det(A^{-1}) = 1/\det(A)$

(C) $\det(A^T) = \det(A)$

(D) rank (A) = order (A), if det(A) = 0

34. If A is an orthogonal matrix, then det(A) is

(A) >0 (B) <0 (C) +1 or -1 (D) 0

35. If V is a vector space and $T: V \to V$ is a linear map then

(A) T^n is a linear map for all positive integers n

(B) T^n is not a linear map if n is a positive integer ≥ 2

(C) T^n is a linear map only if n is an odd positive integer

(D) T^2 is linear but T^3 is not linear.

36. If A is a square matrix of order n and α is a scalar then

(A) $\det(\alpha A) = \alpha \det(A)$ (B) $\det(\alpha A) = \alpha^n \det(A)$

(C) $\det(\alpha A) = \det(A)$ (D) $\det(\alpha A) = |\alpha| \det(A)$

37. If A is a square matrix then

(A) $\det(A^{-1}) = 1/\det(A)$ (B) $\det(A^{-1}) = \det(A)$

(C) $\det(A^{-1}) = -\det(A)$ (D) $\det(A^{-1}) = 1$

38. Let G be an additive group of integers modulo 24. The number of distinct subgroups of G is

(A) 24 (B) 12 (C) 8 (D) 1

39. (A) Every vector space has a finite basis

(B) Every finite vector space need not have a basis

(C) Every finite dimensional vector space has a finite basis

(D) Every infinite dimensional vector space need not have a basis

40. For the set of all n×n matrices the similarity of matrices is

(A) a reflexive but not symmetric relation

(B) a symmetric but not reflexive relation

(C) a transitive but not symmetric relation

(D) an equivalence relation.

41. (A) Any singular matrices can be expressed as a product of elementary matrices

(B) Any singular matrices can be expressed as a sum of elementary matrices

(C) Any non-singular matrices can be expressed as a product of elementary matrices

(D) Any non-singular matrices can be expressed as a sum of elementary matrices

42. The set of points in the complex plane for which $|z-2| + |z+2i| = 4$ is a

(A) Hyperbola (B) Rectangle (C) Square (D) ellipse

43. If $f(z) = z|z|$, for z in the complex plane then f(z) is differentiable

(A) at all point z (B) only at z=0

(C) only at z=1 (D) nowhere

44. The function $w = e^z$, for z in $\mathbf{C}$ is

(A) entire and periodic (B) entire and non periodic

(C) periodic and not entire (D) neither entire nor periodic

45. The function w= cos z, for z in **C** is

(A) unbounded and entire (B) bounded and entire

(C) bounded but nowhere analytic (D) unbounded but nowhere analytic

46. Harmonic conjugate of the function 4xy+3 is

(A) $2(y^2-x^2)$ (B) $-2(y^2-x^2)$

(C) $2(y^2+x^2)$ (D) (y^2-x^2)

47. Log $(-2+2i)^2$ *is*

(A) $2(\log(-1+i)+\log 2)$ (B) $\log 8 - i\frac{\pi}{2}$

(C) 2 log (-2+2i) (D) $2\log(-1+i)$

48. The period of the function sin (13iz), for z in **C** is

(A) $\frac{2\pi i}{13}$ (B) $\frac{\pi i}{13}$ (C) $\frac{\pi i}{7}$ (D) $\frac{4\pi i}{5}$

49. A value of $\sin^{-1}1+\cos^{-1}1$ is given by

(A) 20π (B) 21π (C) $(20 + ½)\ \pi$ (D) $(20 - ½)\ \pi$

50. The branch cut of the function f(z) = (z + 2) log (z-1-i) , $z \in$ **C**, is the set

(A) s= { z=(x,y) : x≤1, y=1} (B) s= {z=(x,y) : x≥1, y=1}

(C) s= {z=(x,y) : x≥1, y≤1} (D) s={ z=(x,y) : x=1, y≥1}

51. The value of the integral $\int_{|z+\pi|=2} \frac{\sin z}{e^z-1}\,dz$ is

(A) zero (B) $2\pi i$ (C) 4π (D) $4\pi i$

52. If P is a polynomial of degree 10 that has 5 distinct roots in an interval (a, b) then the third derivative of P

(A) may not have any roots in (a,b)

(B) must have at least 3 distinct roots in (a,b)

(C) must have at least 2 distinct roots in (a,b)

(D) must have 8 distinct roots in (a,b)

53. The value of $\lim_{x \to 0}\left(\frac{a^x + b^x}{2}\right)^{\frac{1}{x}}$ is

(A) ab (B) $\frac{ab}{2}$ (C) $\sqrt{ab}$ (D) $\frac{a+b}{2}$

54. If $u = \log\left(\frac{x^2 + y^2}{x + y}\right)$ then

(A) $xu_x + yu_y = 1$ (B) $yu_x - xu_y = 1$

(C) $xu_x - yu_y = 1$ (D) $yu_x + xu_y = 1$

55. The radius of a circle was measured as 50 cm. Then the percentage error in the area of the circle, due to an error of 1mm. while measuring the radius is

(A) 0.2 (B) 0.4 (C) 0.1 (D) 0.5

56. The n^{th} derivative of y=sin (ax+b) is

(A) $a^n \, Sin\left(n\frac{\pi}{2} + ax + b\right)$ (B) $b^n \, Sin\left(n\frac{\pi}{2} + ax + b\right)$

(C) $(a+b)^n \, Sin(n\pi + ax + b)$ (D) $a^n \, Sin(n\pi + ax + b)$

57. (A) Every bounded sequence is convergent

(B) Every monotonic sequence is convergent

(C) Every sequence has a convergent subsequence

(D) Every sequence has a monotone subsequence

58. The Laplace transform of $f(x) = x^2 e^{-3x}$ is

(A) $\frac{2}{(s+3)^3}$ (B) $\frac{1}{(s+3)^2}$ (C) $\frac{2}{(s-3)^3}$ (D) $\frac{1}{s^2+3^2}$

59. $\lim_{n \to \infty} \frac{1}{n}\left(1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}\right)$ equals

(A) 0 (B) e (C) 1 (D) ∞

60. Mark the **wrong** statement.

(A) $\left(\frac{1}{n}\right)_{n=1}^{\infty}$ is a Cauchy sequence

(B) $(n)_{n=1}^{\infty}$ is not a Cauchy sequence

(C) $\left(\frac{(-1)^n}{n}\right)_{n=1}^{\infty}$ is a Cauchy sequence

(D) $\left((-1)^n + \frac{1}{n}\right)_{n=1}^{\infty}$ is a Cauchy sequence

61. The function $f(z) = |z|^2$, $z \in \mathbf{C}$, is

(A) differentiable everywhere

(B) differentiable only at the origin

(C) not differentiable anywhere

(D) differentiable only on the real axis

62. The fixed points of $f(z) = \frac{2iz+5}{z-2i}$ are

(A) $1 \pm i$

(B) $1 \pm 2i$

(C) $2i \pm 1$

(D) $i \pm 1$

63. The value of $\oint_{|z|=1} \frac{e^z}{(z-2)(z-3)} dz$ is

(A) $2\pi i$

(B) πi

(C) 0

(D) $\frac{\pi i}{6}$

64. The value of the constant a for which the function $u(x, y) = ax^2 - y^2 + xy$ is harmonic is

(A) 1

(B) 5

(C) 0

(D) -1

65. $\lim_{x \to \infty} x^2 e^{-x} =$

(A) 0

(B) 1

(C) ∞

(D) does not exist

66. Let $f: R \to R$ be a continuous function. Then the set $\{x \in R | f(x) = 0\}$ is a

(A) compact subset of R

(B) open subset of R

(C) closed subset of R

(D) connected subset of R

67. The value of $\int_{-2}^{2} (1 - |x|)\, dx$ is

(A) 1

(B) -1

(C) $\frac{1}{2}$

(D) 2

68. If $u(x,y) = e^{\frac{-y}{x}}\left(\cos\left(\frac{y}{x}\right) + \sin\left(\frac{y}{x}\right)\right)$ then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} =$

(A) $x^2 + 2xy$

(B) $x^2 + 2xy^2 + \sin\left(\frac{y}{x}\right)$

(C) $x^2y + 2xy^2 + \cos\left(\frac{y}{x}\right)$

(D) 0

69. $\lim_{n\to\infty}\left(\sqrt{n^2+n} - n\right) =$

(A) ∞ (B) 1 (C) $\frac{1}{2}$ (D) $\frac{-1}{2}$

70. If $f:[0,1]\to[0,1]$ is defined as $f(x) = \begin{cases} 1 & if\ x\ is\ rational \\ 0 & if\ x\ is\ irrational \end{cases}$ Then

(A) f is continuous at all points of $[0,1]$

(B) f is continuous only at the rational points of $[0,1]$

(C) f is continuous only at the irrational points of $[0,1]$

(D) f is continuous nowhere in $[0,1]$

71. The value of the integral $\int_{-\pi}^{\pi} x^{100}\sin x\ dx$ is

(A) $\frac{\pi^{101}}{101}$ (B) 0 (C) $\frac{2\pi^{101}}{101}$ (D) $\frac{-2\pi^{101}}{101}$

72. If $f(z) = \frac{1}{\sin\frac{1}{z}}$, $z \neq 0$, then

(A) $z = \pi$ is a pole of f

(B) $z = \frac{1}{6\pi}$ is a pole of f

(C) $z = \frac{1}{\pi}$ is a pole of f

(D) f has no poles

73. If λ is an eigenvalue of an unitary matrix then

(A) $\lambda = 1$ (B) $\lambda = 0$ (C) λ is real (D) $|\lambda| = 1$

74. Mark the **wrong** statement

(A) Any cyclic group is abelian

(B) Any abelian group is cyclic

(C) Any abelian group satisfies $(ab)^2 = a^2b^2$

(D) Any subgroup of an abelian group is normal

75. The order of any non zero element a in the group $(\mathbf{Z},+)$ is

(A) 2 (B) finite (C) 1 (D) infinite

76. The Maclaurin Series expansion of $f(z) = \frac{z}{z^4+9}$ is

(A) $\sum_{n=0}^{\infty} \frac{1}{2^{n+1}} z^{4n+z}$, $|z|<2$

(B) $\sum_{n=0}^{\infty} \frac{(-1)^n}{3^{n+1}} z^{4n+1}$, $|z|<\sqrt{3}$

(C) $\sum_{n=0}^{\infty} \frac{1}{3^{2n+1}} z^{4n+1}$, $|z|<4$

(D) $\sum_{n=0}^{\infty} \frac{(-1)^n}{4^{n+2}} z^{4n}$, $|z|<4$

77. If $f_n(x)= x^n, x \in [0,1]$, for any positive integer n, then the sequence (f_n) of functions

(A) Does not converge point wise at any point in $[0,1]$

(B) Converges uniformly in $[0,1]$

(C) Converges pointwise only on a countable subset of $[0,1]$

(D) Converges pointwise but not uniformly on $[0,1]$

78. If f(z)= Re z, for any complex number z, then

(A) f is differentiable at all points of the complex plane

(B) f is continuous at all points of the complex plane but differentiable only at z=0.

(C) f is not differentiable at any point of the complex plane but continuous at all points of the complex plane.

(D) f is discontinuous

79. If $f(z)= \frac{1-\cos z}{z^2}$, then z=0 is

(A) a double pole (B) a simple pole

(C) is a removable singularity (D) is an essential singularity

80. The value of the line integral $\int x \, dz$ along the straight line joining (0,0) and (1,1) is

(A) $\frac{1}{2}+i\frac{1}{2}$ (B) $\frac{1}{2}-i\frac{1}{2}$ (C) 0 (D) $\frac{1}{2}$

81. Let $f: G \to H$ is a group homomorphism. Then G is isomorphic to f(G)

(A) if and only if f is onto (B) if and only if f is 1-1

(C) only if f is an isomorphism (D) if G is cyclic

82. Let R be a ring and a,b be invertible elements of R. Then the product,

(A) ab is invertible

(B) ab is invertible if R is commutative ring

(C) ab is invertible but ba need not be invertible

(D) ab is invertible if R is a field

83. The eigenvalues of the matrix $A = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$ are

(A) 1 and -i

(B) i and -i

(C) 1+i and 1-i

(D) 1- 2i and 1+2i

84. If $f(x) = x^3 - 3x^2 - 9x + 5$, then in the interval [-2, 2], f has

(A) one maxima and no minima

(B) has two minima

(C) has two maxima

(D) one minima and no maxima

85. If the eigen values of $A = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{pmatrix}$ are -2, 3, 6, then the eigen values of the transpose A^T are...

(A) $-2,3,6$

(B) $\frac{1}{2},\frac{1}{3},\frac{1}{6}$

C) $-2^2,3^2,6^2$

(D) $-4,6,12$

86. The sum of the characteristic roots of $\begin{pmatrix} \cos\theta & -\sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$ are....

(A) 0

(B) 1

(C) $2\cos\theta$

(D) $2\sin\theta$

87. The area under the curve $y = \frac{1}{x}$ between the ordinates at $x = 1$ *and* $x = 2$ *is*

(A) $\frac{1}{2}$

(B) $\log 2$

(C) $e-1$

(D) 1

88. A particular solution of the equation $y'' + y = \sin x$ is given by

(A) $\sin x$

(B) $\cos x$

(C) $\frac{-x\cos x}{2}$

(D) $\frac{-x\sin x}{2}$

89. If $\vec{x}, \vec{y}$ *and* $\vec{z}$ are 3 non zero vectors in $\mathbf{R}^3$ then $(\vec{x}\times\vec{y}).\vec{z}=0$ implies

(A) $\vec{z}$ is parallel to $\vec{x}\times\vec{y}$

(B) $\vec{z}$ lies in the plane containing $\vec{x}$ and $\vec{y}$

(C) $\vec{z}$ is perpendicular to the plane containing $\vec{x}$ and $\vec{y}$

(D) Either $\vec{x}\,.\,\vec{z}=0$ or $\vec{y}\,.\,\vec{z}=0$

90. If $A=\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}$ and $V=\{Ax : x\in\mathbf{R}^2\}$ then

(A) V is a one dimensional vector space

(B) $V=\mathbf{R}^2$

(C) $V=\{0\}$

(D) V is not a vector space

91. The set integers is not a group under the binary operator minus, because

(A) associativity property is not satisfied

(B) closure property is not satisfied

(C) inverse with respect to minus operator does not exist for each integer

(D) commutative property is not satisfied

92. The order of the smallest non abelian group is

(A) 4 (B) 5 (C) 6 (D) 8

93. Let X and Y be sets with cardinalities m and n respectively. If the number of possible functions that can be defined with domain X and co domain Y is exactly 10, then

(A) m=n=10 (B) m=1; n=10 (C) m=10; n=1 (D) m=5; n=5

94. The total number of equivalence relations that can be defined on the set {1,2,3} is

(A) 8 (B) 64 (C) 5 (D) 3

95. Recall that the set $\{1, w, w^2\}$ of cube roots of unity forms a cyclic group under multiplication.

For this group,

(A) w is the only generator (B) w^2 is the only generator

(C) both w and w^2 are generators (D) neither w nor w^2 is a generator.

96. Let the characteristic equation of a matrix M be $\lambda^2 - \lambda - 1 = 0$, *then*

(A) M^{-1} does not exist

(B) M^{-1} exists but cannot be determined from the data

(C) $M^{-1} = M + 1$

(D) $M^{-1} = M - 1$

97. If 0 is an eigenvalue of a square matrix A then

(A) det A $\neq$ 0

(B) A^{-1} does not exist

(C) A is symmetric

(D) A is diagonal

98. The characteristic equation of $\begin{pmatrix} -m & -n \\ 1 & 0 \end{pmatrix}$ *is*

(A) $x^2 - mx - n$

(B) $x^2 + mx + n$

(C) $x^2 + nx + m$

(D) $x^2 + nx + mn$

99. If A is the 2×2 matrix $\begin{bmatrix} \lambda & 1 \\ 0 & \lambda \end{bmatrix}$ where $\lambda \in \mathbf{R}$, then A^n is

(A) $\begin{bmatrix} \lambda^n & 1 \\ 0 & \lambda \end{bmatrix}$

(B) $\begin{bmatrix} \lambda & 1 \\ 0 & \lambda^n \end{bmatrix}$

(C) $\begin{bmatrix} \lambda^n & n\lambda^{n-1} \\ 0 & \lambda^n \end{bmatrix}$

(D) $\begin{bmatrix} n\lambda^{n-1} & \lambda^n \\ 0 & n\lambda^{n-1} \end{bmatrix}$

100. The radius of convergence of the Power series $\sum_{k=1}^{\infty} \frac{z^k}{k^2}$ is

(A) 1

(B) 2

(C) 0

(D) ∞