

ENTRANCE EXAMINATION FOR ADMISSION, MAY 2013.

M.Sc. (MATHEMATICS)

COURSE CODE : 372

Register Number :

*Signature of the Invigilator
(with date)*

COURSE CODE : 372

Time : 2 Hours

Max : 400 Marks

Instructions to Candidates :

1. Write your Register Number within the box provided on the top of this page and fill in the page 1 of the answer sheet using pen.
2. Do not write your name anywhere in this booklet or answer sheet. Violation of this entails disqualification.
3. Read each question carefully and shade the relevant answer (A) or (B) or (C) or (D) in the relevant box of the ANSWER SHEET using HB pencil.
4. Avoid blind guessing. A wrong answer will fetch you -1 mark and the correct answer will fetch 4 marks.
5. Do not write anything in the question paper. Use the white sheets attached at the end for rough works.
6. Do not open the question paper until the start signal is given.
7. Do not attempt to answer after stop signal is given. Any such attempt will disqualify your candidature.
8. On stop signal, keep the question paper and the answer sheet on your table and wait for the invigilator to collect them.
9. Use of Calculators, Tables, etc. are prohibited.

1. In a group G , if a is not an identity element then which one is not possible:

(A) $a^2 b = b$	(B) $a^2 b = ab$
(C) $aba = a$	(D) $ab^2 = b$
2. If K is a subgroup of H and H is a subgroup of a finite group G under same binary operation, then

(A) $o(K)$ divides both $o(H)$ and $o(G)$.	(B) $o(K)$ divides $o(G)$ but $o(K)$ need not divide $o(H)$
(C) $o(K)$ divides $o(H)$ but $o(K)$ need not divide $o(G)$	(D) $o(K)$ need not divide $o(G)$ and $o(K)$ need not divide $o(H)$
3. If a, b are in a group G , a not equal to b and H be a non trivial subgroup of G then which one is not possible

(A) $aH \cap bH = \{e\}$	(B) $aH \cap bH = \emptyset$
(C) $aH \cap bH = aH$	(D) a not equal to e and $aH = H$
4. A group G is abelian if for every a, b in G

(A) $a^2 b^2 = a(ab)b$	(B) $(ab)^2 = abab$
(C) $a^2 = e$ and $b^2 = e$	(D) $aabb = a^2 b^2$
5. The binary operator on a set S is

(A) a subset of $S \times S$	(B) a function from S to R , the set of real numbers
(C) A function from S to S	(D) a function from $S \times S$ to S
6. Which of the following property is satisfied in a group G ?
For every a, b in G

(A) $(ab)^2 = a^2 b^2$	(B) for c in G $ab = ac$ implies $b = c$
(C) $aba^{-1} = b$	(D) $a^2 = e$ implies $a = e$
7. The number of distinct left cosets of a subgroup H in a finite group G is

(A) $o(H)$	(B) $o(G) - o(H)$
(C) $o(H) + 1$	(D) $o(G)/o(H)$
8. If N is a normal subgroup of G , then

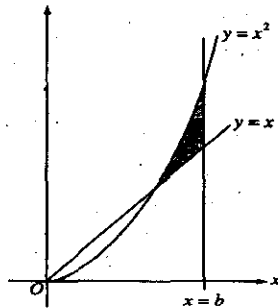
(A) $Na = Nb$ for every a, b in G	(B) $an = na$ for every a in G and n in N
(C) $nm = mn$ for every n, m in N	(D) ana^{-1} belong to N for every n in N and a in G
9. Let G be a group and H be subgroup of G . Then G/H is well defined quotient group if

(A) H is an abelian subgroup of G	(B) H is a normal subgroup of G
(C) H is cyclic subgroup of G	(D) H is a finite subgroup of G
10. If the order of a quotient group G/H is finite then

(A) G is finite	(B) H is finite
(C) $G \setminus H$ is finite	(D) None of the above

11. If $F(x) = \int_{\frac{1}{e}}^x \log t \, dt$ for all positive x , then $F'(x) =$
- (A) x (B) $1/x$
 (C) $\log x$ (D) $x \log x$

12.



If $b > 0$ and if $\int_0^b x \, dx = \int_0^b x^2 \, dx$, then the area of the shaded region in the figure above is

- (A) $1/12$ (B) $1/6$
 (C) $1/4$ (D) $1/3$
13. If $F(1) = 2$ and $F(n) = F(n-1) + 1/2$ for all integers $n > 1$, then $F(101) =$
- (A) 49 (B) 50
 (C) 51 (D) 52
14. Let g be the function defined on the set of all real numbers by $\begin{cases} 1 & \text{if } x \text{ is rational} \\ e^x & \text{if } x \text{ is irrational} \end{cases}$. Then the set of all numbers at which g is continuous is
- (A) The empty set (B) $\{0\}$
 (C) $\{1\}$ (D) The set of rational numbers
15. If S is a nonempty finite set with K elements, then the number of one-to-one functions from S to S is
- (A) $K!$ (B) K^2
 (C) K^K (D) 2^K
16. Let R be the set of real numbers and let f and g be functions from R into R . The negation of the statement "For each s in R , there exist an r in R such that if $f(r) > 0$, then $g(s) > 0$ " is which of the following
- (A) For each s in R , there does not exist an r in R such that if $f(r) > 0$, then $g(s) > 0$
 (B) For each s in R , there exist an r in R such that $f(r) > 0$ and $g(s) \leq 0$
 (C) There exist an s in R such that for each r in R , $f(r) > 0$ and $g(s) \leq 0$
 (D) There exist an s in R and, there exist an r in R such that $f(r) \leq 0$ and $g(s) \leq 0$
17. The Expansion $\sum_{n=0}^{\infty} z^n/3^n + \sum_{n=1}^{\infty} 1/z^n$, $1 < |z| < 3$, then $z=0$ is
- (A) Essential singularity (B) Simple pole
 (C) Regular point (D) none

18. Which of the following statements are true for every function f , defined on the set of all real number, such that $\lim_{x \rightarrow 0} f(x)/x$ is a real number L and $f(0)=0$?
- I f is differentiable at 0
 II $L=0$
 III $\lim_{x \rightarrow 0} f(x)=0$
- (A) I only (B) III only
 (C) I and III only (D) I, II and III only
19. If g is a function defined on the open interval (a,b) such that $a < g(x) < x$ for all $x \in (a,b)$, then g is
- (A) An unbounded function (B) A non constant function
 (C) A non negative function (D) A strictly increasing function
20. Let Z be the group of all integers under the operation of addition. Which of the following subsets of Z is NOT a subgroup of Z ?
- (A) $\{0\}$
 (B) $\{n \in Z: n \geq 0\}$
 (C) $\{n \in Z: n \text{ is an even integer}\}$
 (D) $\{n \in Z: n \text{ is divisible by both 6 and 9}\}$
21. Let f be a function such that $f(x)=f(1-x)$ for all real numbers x . If f is differentiable everywhere, then $f'(0)=$
- (A) $f(0)$ (B) $f(1)$
 (C) $f'(1)$ (D) $-f'(1)$
22. If V_1 and V_2 are 6 dimensional subspaces of a 10- dimensional vector space V , what is the smallest possible dimension that $V_1 \cap V_2$ can have ?
- (A) 0 (B) 2
 (C) 4 (D) 6
23. Suppose B is a basis for a real vector space V of dimension greater than 1. Which of the following statements could be true?
- (A) The zero vector of V is an element of B .
 (B) B has a proper subset that spans V
 (C) B is a proper subset of a linearly independent subset of V .
 (D) There is a basis for V that is disjoint from B .
24. How many integers from 1 to 1000 are divisible by 30 but not by 16?
- (A) 28 (B) 31
 (C) 32 (D) 33
25. $\sum_{k=1}^{\infty} \frac{2^k}{k!} =$
- (A) E (B) $2e$
 (C) $(e+1)(e-1)$ (D) e^2

26. A fair coin is to be tossed 8 times. What is the probability that more of the tosses will result in heads than will result in tails?
- (A) $1/4$ (B) $1/3$
 (C) $93/256$ (D) $23/64$
27. Let X and Y be uniformly distributed, independent random variables on $[0,1]$. The probability that the distance between X and Y is less than $\frac{1}{2}$ is
- (A) $1/4$ (B) $1/3$
 (C) $1/2$ (D) $3/4$
28. In the complex plane, let C be the circle $|z| = 2$ with positive orientation. Then the integral

$$\int_C \frac{dz}{(z-1)(z+3)^2}$$
 is

- (A) 0 (B) $2\pi i$
 (C) $\pi i/2$ (D) $\pi i/8$
29. What is the greatest integer that divides $p^4 - 1$ for every prime number p greater than 5?
- (A) 20 (B) 30
 (C) 120 (D) 240

30. The general solution of the equation $\frac{d^2 y}{dx^2} - 6\frac{dy}{dx} + 13y = 0$ is

- (A) $e^x(c_1 x^2 + c_2 x + c_3)$ (B) $e^{3x}(c_1 \cos 2x + c_2 \sin 2x)$
 (C) $e^x(c_1 x^2 + x(c_2 + c_3))$ (D) $e^{2x}(c_1 \sin^2 x + c_2 \cos x + c_3)$

31. The solution of the equation $\frac{d^5 y}{dx^5} - \frac{d^3 y}{dx^3} = 0$ is

- (A) $c_1 e^{2x} + c_2 e^x + (c_3 + c_4 x + c_5 x^2)e^{0 \cdot x}$ (B) $c_1 e^{-x} + c_2 e^x$
 (C) $c_1 e^{-x} + c_2 e^x + (c_3 + c_4 x + c_5 x^2)e^{3 \cdot x}$ (D) $c_1 e^{-x} + c_2 e^x + (c_3 + c_4 x + c_5 x^2)e^{0 \cdot x}$

32. $\int_{-a}^a x^3 \sqrt{a^2 - x^2} dx =$

- (A) a^3 (B) $2a^3$
 (C) 0 (D) $-2a^3$

33. $\int \log x dx =$

- (A) $1/\log x$ (B) $2 \log x - x$
 (C) 0 (D) $x \log x - x$

34. If $f(2a-x) = -f(x)$, then $\int_0^{2a} f(x) dx =$

(A) $2a$

(B) a

(C) 0

(D) a^2

35. $\int_0^{\pi/2} \frac{\sin x}{\cos x + \sin x} dx =$

(A) $\pi/4$

(B) $\pi/2$

(C) π

(D) 0

36. If L denotes the Laplace transform, then $L^{-1} \left[\frac{1}{(s+3)^4} \right] =$

(A) e^{-3t}

(B) $t^3 e^{-3t}$

(C) $\frac{t^3}{3!} e^{-3t}$

(D) $\frac{t^3}{3!} e^{3t}$

37. If matrix A is $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ -2 & -3 & -1 \end{bmatrix}$, then the rank of A is

(A) 2

(B) 1

(C) 3

(D) 0

38. $\frac{\sqrt{2} + i\sqrt{2}}{2} =$

(A) $e^{\frac{\pi}{4}}$

(B) $e^{\frac{\pi}{4}}$

(C) $e^{\frac{\pi}{4}}$

(D) $e^{\frac{\pi}{3}}$

39. $\cosh 2x =$

(A) $\frac{1 - \tanh^2 x}{1 + \tanh^2 x}$

(B) $\frac{1 + \tanh^2 x}{1 - \tanh^2 x}$

(C) $\frac{2 - \tanh^2 x}{2 + \tanh^2 x}$

(D) $\frac{1 + \tanh^2 x}{1 - \tanh^2 x}$

40. The trace of a real square matrix is equal to

(A) 0

(B) 1

(C) product of its eigen values

(D) sum of its eigen values

41. Which one of the following statements is correct ?
- (A) Any two cyclic groups of the same orders are isomorphic
 (B) Any two abelian groups of the same order are isomorphic
 (C) Any two finite groups of the same order are isomorphic
 (D) Any two infinite groups are isomorphic.
42. The union of two subgroups H and K of a group G is a subgroup if one of the following holds:
- (A) G is abelian (B) H and K are disjoint
 (C) $H \subseteq K$ or $K \subseteq H$ (D) H and K are finite
43. $\lim_{n \rightarrow \infty} n \cdot \log\left(1 + \frac{1}{n}\right)$ is
- (A) 0 (B) 1
 (C) ∞ (D) n
44. Which of the following real-valued functions on (0, 1) is uniformly continuous ?
- (A) $f(x) = \frac{1}{x}$ (B) $f(x) = \sin\left(\frac{1}{x}\right)$
 (C) $f(x) = \left(\frac{\sin x}{x}\right)$ (D) $f(x) = \left(\frac{\cos x}{x}\right)$
45. The system of equations $x + y + z = 1$; $2x + 3y - z = 5$ and $x + 2y - kz = 4$, where $k \in \mathbf{R}$, has an infinite number of solutions for
- (A) $k = 0$ (B) $k = 1$
 (C) $k = 2$ (D) $k = 3$
46. Let $f(x)$ be the minimal polynomial of the (4×4) - matrix A, where $A = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$.
- The rank of the (4×4) - matrix $f(A)$ is
- (A) 0 (B) 1
 (C) 2 (D) 4
47. What is the next number in the sequence. 30, 33, 39, 51.....?
- (A) 54 (B) 57
 (C) 9 (D) 75

48. The number $\sqrt{2} \cdot e^{i\pi}$ is a
- (A) Rational number (B) Transcendental number
 (C) Irrational number (D) Imaginary number
49. The number of nontrivial ring-homomorphisms from $\mathbf{Z}_{(12)}$ to $\mathbf{Z}_{(28)}$ is
- (A) 1 (B) 3
 (C) 4 (D) 7
50. Let A and B be $(n \times n)$ -real matrices. Which of the following statements is correct?
- (A) $\text{Rank}(A+B) = \text{Rank}(A) + \text{Rank}(B)$
 (B) $\text{Rank}(A+B) \leq \text{Rank}(A) + \text{Rank}(B)$
 (C) $\text{Rank}(A+B) = \min\{\text{Rank}(A), \text{Rank}(B)\}$
 (D) $\text{Rank}(A+B) = \max\{\text{Rank}(A), \text{Rank}(B)\}$
51. If $f(x) = ax^3 - 9x^2 + 9x + 3$ is an increasing function on \mathbf{R} , then which one of the following is true?
- (A) $a = 3$ (B) $a < 3$
 (C) $a > 3$ (D) $a \leq 3$
52. An ideal A of a commutative ring R with unity is maximal if and only if R/A is
- (A) Integral domain (B) division ring
 (C) field (D) none of these
53. If A is a $(n \times n)$ complex matrix with $A^2 = 0$, then the eigen values of A are
- (A) 0 (B) 1
 (C) 0 and 1 (D) 1 and 2A
54. The radius of convergence of the power series $\sum_{n=1}^{\infty} (n!) \cdot Z^n$ is equal to
- (A) 0 (B) $\frac{1}{2}$
 (C) 1 (D) ∞
55. If $\cos \alpha, \cos \beta, \cos \gamma$ are the direction cosines of a straight line, then $\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma =$
- (A) 1 (B) 0
 (C) 2 (D) 1/2

56. The equation of the line through (1,1,1) and perpendicular to the plane $x-y-z-10=0$ is
- (A) $\frac{x-1}{1} = \frac{y-1}{1} = \frac{z-1}{1}$
 (B) $\frac{x-1}{1} = \frac{y+1}{1} = \frac{z+1}{-1}$
 (C) $\frac{x-1}{1} = \frac{y-1}{-1} = \frac{z-1}{-1}$
 (D) $\frac{x-1}{1} = \frac{y+1}{1} = \frac{z+1}{1}$
57. The equation of a plane passing through the line of intersection of the planes $2x-y+5z-3=0$ and $4x+2y-z+7=0$ and parallel to the z -axis is
- (A) $2x+3y=12$ (B) $8x-2y-21=0$
 (C) $22x+9y+32=0$ (D) $x+y-10=0$
58. The distance between the point (x_1, y_1, z_1) and the plane $ax+by+cz+d=0$ is given by
- (A) $ax_1 + by_1 + cz_1 + d$ (B) $\pm \frac{ax_1 + by_1 + cz_1 + d}{\sqrt{a^2 + b^2 + c^2}}$
 (C) $\frac{ax_1 + by_1 + cz_1 + d}{a^2 + b^2 + c^2}$ (D) $\sqrt{a^2 + b^2 + c^2}$
59. The condition for a straight line $\frac{x-x_1}{l} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$ to lie on a plane $ax+by+cz+d=0$ is
- (A) $al + bm + cn = 0$ and $ax_1 + by_1 + cz_1 + d = 0$
 (B) $al + bm + cn = 0$ and $ax_1 + by_1 + cz_1 + d \neq 0$
 (C) $al + bm + cn = 0$
 (D) $ax_1 + by_1 + cz_1 + d = 0$
60. The equation of the sphere having the circle $x^2 + y^2 + z^2 + 10y - 4z - 8 = 0; x + y + z - 3 = 0$ as a great circle is
- (A) $x^2 + y^2 + z^2 - 4x + 6y - 8z + 4 = 0$ (B) $x^2 + y^2 + z^2 + x + 11y - 3z - 11 = 0$
 (C) $x^2 + y^2 + z^2 + 4x - 6y - 8z + 4 = 0$ (D) $x^2 + y^2 + z^2 - x + 9y - 5z - 5 = 0$
61. $\sinh 3x =$
- (A) $3\sinh x - 4\sinh^3 x$ (B) $4\sinh x - 3\sinh^3 x$
 (C) $3\sinh x + 4\sinh^3 x$ (D) $4\sinh x + 3\sinh^3 x$

62. $\lim_{\theta \rightarrow 0} \frac{\tan \theta - \sin \theta}{\theta^3} =$
- (A) 1 (B) 0 (C) 2 (D) 1/2
63. In how many ways can ten red balls, ten white balls and ten blue balls be placed in 50 different boxes, at most one ball to a box?
- (A) $\binom{50}{30}$ (B) $3! \binom{50}{30}$
- (C) $\binom{50}{10} \binom{40}{10} \binom{30}{10}$ (D) $\frac{50!}{3!(10!)^3 20!}$
64. $\int_C \frac{1}{2z+3} dz$ where C is $|Z|=2$ is
- (A) $2\pi i$ (B) πi (C) $4\pi i$ (D) 1
65. For the function $\frac{e^z}{z^2+4}$
- (A) $z=0$ is a simple pole
 (B) $z=0$ is a removable singularity
 (C) $z=0$ is a removable singularity and $z=2i$ is the only simple pole
 (D) $z=\pm 2i$ are simple pole
66. If H and K are finite subgroups of G of order $o(H)$ and $o(K)$ then
- (A) $o(HK) = \frac{o(H)+o(K)}{o(H \cap K)}$ (B) $o(HK) = \frac{o(H)o(K)}{o(H \cap K)}$
- (C) $o(HK) = \frac{o(H)+o(K)}{o(H \cup K)}$ (D) $o(HK) = \frac{o(H)o(K)}{o(H \cup K)}$
67. If $\kappa(G)$ and $\kappa'(G)$ denotes the vertex connectivity and edge connectivity of a graph G then $\kappa(G) = \kappa'(G)$ if
- (A) G is a simple graph with even degree vertices.
 (B) G is a simple graph whose maximum degree is at most 4.
 (C) G is a simple 3-regular graph.
 (D) G is Hamiltonian.

68. Which of the following is false?
- (A) If a group G has exactly one subgroup H of given order then H is a normal subgroup of G .
- (B) If H is a subgroup of G and N is a normal subgroup of G then HN is a subgroup of G .
- (C) If $f: G \rightarrow G'$ is a homomorphism then f is bijection iff $\text{Ker } f = \{e\}$.
- (D) Any unit in R cannot be a zero divisor.
69. If G is a k -critical graph then which of the following is not true.
- (A) Maximum degree of the graph is at most $k-1$.
- (B) Minimum degree of the graph is at least $k-1$.
- (C) Any vertex cut does not form a clique.
- (D) $\chi(G-v) \leq \chi(G)-1$ for every vertex of G .
70. $\lim_{n \rightarrow \infty} \frac{1}{n^2} \left(1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n} \right) =$ _____
- (A) 0 (B) e (C) 1 (D) ∞
71. If $a_n = \frac{10^7}{n}$ and $b_n = \frac{n}{10^7}$, for $n = 1, 2, 3, \dots$, then which of the following is true?
- (A) $(a_n)_{n=1}^{\infty}$ is convergent and $(b_n)_{n=1}^{\infty}$ is divergent.
- (B) $(a_n)_{n=1}^{\infty}$ is divergent and $(b_n)_{n=1}^{\infty}$ is convergent.
- (C) Both $(a_n)_{n=1}^{\infty}$ and $(b_n)_{n=1}^{\infty}$ are convergent.
- (D) Both $(a_n)_{n=1}^{\infty}$ and $(b_n)_{n=1}^{\infty}$ are divergent.
72. If the sequence $(a_n)_{n=1}^{\infty}$ is defined as $a_1 = -1$, $a_2 = 0$ and $a_n = 1$, if $n > 2$, then
- (A) $\text{LimSup } a_n = 1$ and $\text{Lim inf } a_n = -1$.
- (B) $\text{LimSup } a_n = 1$ and $\text{Lim inf } a_n = 1$
- (C) $\text{LimSup } a_n = 0$ and $\text{Lim inf } a_n = -1$.
- (D) $\text{LimSup } a_n = 1$ and $\text{Lim inf } a_n = 0$.
73. Consider the following series $\sum_{n=1}^{\infty} a_n = 1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \dots$ and $\sum_{n=1}^{\infty} b_n = 1 - 1 + \frac{1}{2} - \frac{1}{2} + \frac{1}{3} - \frac{1}{3} + \frac{1}{4} - \frac{1}{4} + \dots$. Which of the following is true?
- (A) $\sum_{n=1}^{\infty} a_n$ is convergent and $\sum_{n=1}^{\infty} b_n$ is divergent.
- (B) $\sum_{n=1}^{\infty} a_n$ is divergent and $\sum_{n=1}^{\infty} b_n$ is convergent.
- (C) Both $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are convergent.
- (D) Both $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are divergent.
74. The radius of convergence of the power series $\sum_{n=1}^{\infty} n^{10,000} x^n$ is
- (A) 0 (B) 10,000 (C) 1 (D) $\frac{1}{2}$

75. If $f(t) = \begin{cases} e^{-t} & \text{if } 0 < t < 4 \\ 0 & \text{if } t \geq 4 \end{cases}$ then the Laplace transform of $f(t)$ is given by
- (A) $\frac{1 - e^{-4(s+1)}}{s+1}$ (B) $\frac{1 - e^{-4(s-2)}}{s-1}$
 (C) $\frac{1 - e^{-4(s+1)}}{(s+1)^2}$ (D) $\frac{1 - e^{-4(s+1)}}{(s-1)^2}$
76. Which of the following function is uniformly continuous ?
 (A) $f: \mathbb{R} \rightarrow \mathbb{R}$ defined as $f(x) = x^2$.
 (B) $f: (0,1) \rightarrow \mathbb{R}$ defined as $f(x) = \frac{1}{x}$
 (C) $f: (0,1) \rightarrow \mathbb{R}$ defined as $f(x) = \sin\left(\frac{1}{x}\right)$
 (D) $f: (0,1) \rightarrow \mathbb{R}$ defined as $f(x) = e^x$
77. Which of the following pairs of metric spaces (with usual metric) are homeomorphic?
 (A) $(0,1)$ and $(0,2)$ (B) $(0,4)$ and $[0,4]$
 (C) $[0, \infty)$ and $(1, \infty)$ (D) $[0,1]$ and \mathbb{R} .
78. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ and $g: \mathbb{R} \rightarrow \mathbb{R}$ be continuous functions. Let $A = \{x \in \mathbb{R} \mid f(x) = g(x)\}$. Then
 (A) A is closed. (B) A is open
 (C) A is connected. (D) A is compact.
79. Which of the following is a compact subset of \mathbb{R}^2 .
 (A) $\{(x,y) \mid x^2 + y^2 = 1\}$ (B) $\{(x,y) \mid x = 0\}$
 (C) $\{(x,y) \mid x = 0 \text{ or } y = 0\}$ (D) $\{(x,y) \mid x, y \in \mathbb{Q}\}$
80. In \mathbb{R} , define a binary operation $*$ by, $a * b = 1 + ab$. Then
 (A) $*$ is commutative but not associative.
 (B) $*$ is not commutative but associative.
 (C) $*$ is both commutative and associative
 (D) $*$ is neither commutative nor associative
81. Consider the map $f: \mathbb{R} \rightarrow \mathbb{R}$ defined as $f(t) = \begin{cases} 1 & \text{if } x \text{ is rational} \\ 0 & \text{if } x \text{ is irrational} \end{cases}$
 Which of the following is true?
 (A) f is continuous only at the rational points in \mathbb{R} .
 (B) f is continuous only at the irrational points in \mathbb{R} .
 (C) f is continuous everywhere.
 (D) f is continuous nowhere.
82. Suppose $f: \mathbb{R} \rightarrow \mathbb{R}$ is defined as $\begin{cases} -1 & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ 1 & \text{if } x > 0 \end{cases}$. Then
 (A) $\lim_{x \rightarrow 0} f(x) = -1$ (B) $\lim_{x \rightarrow 0} f(x) = 0$
 (C) $\lim_{x \rightarrow 0} f(x) = 1$ (D) $\lim_{x \rightarrow 0} f(x)$ does not exist.

83. The value of the integral $\int_0^{\infty} e^{-x^2} dx$ is
 (A) 0 (B) $\frac{\pi}{2}$ (C) π^2 (D) $\frac{\sqrt{\pi}}{2}$
84. For every set S and every metric d on S, which of the following is a metric on S?
 (A) 4+d (B) $d-|d|$ (C) d^2 (D) \sqrt{d}
85. Let f(z) be an even analytic function having a pole of order two at z=a then the residue of f(z) at z=a is
 (A) -1 (B) 0 (C) 1 (D) 2
86. In the group of all 2×2 non singular matrices over R under matrix multiplication the inverse of $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is
 (A) $\frac{1}{|A|} \begin{pmatrix} a & -b \\ -c & d \end{pmatrix}$ (B) $\frac{1}{|A|} \begin{pmatrix} a & b \\ c & d \end{pmatrix}$
 (C) $\frac{1}{|A|} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$ (D) $-\frac{1}{|A|} \begin{pmatrix} a & -b \\ -c & d \end{pmatrix}$
87. In Z define a binary operation by $a * b = \text{maximum}(a, b)$. Then
 (A) Identity element exists (B) $(Z, *)$ is a group
 (C) $(Z, *)$ is an abelian group (D) $(Z, *)$ is not a group.
88. G is a group with identity e. Then the solution of the equation $xab = c$ for x is
 (A) $x = ca^{-1}b^{-1}$ (B) $x = cb^{-1}a^{-1}$
 (C) $x = a^{-1}b^{-1}c$ (D) $x = b^{-1}a^{-1}c$.
89. The order of -1 in $(R \setminus \{0\}, \cdot)$ is
 (A) 2 (B) 1 (C) -2 (D) infinite.
90. If G is a finite group and H is any subgroup of G then the order of H divides the order of G. This theorem is known as
 (A) Cayley's Theorem (B) Lagrange's theorem
 (C) Euler's Theorem (D) Fermat Theorem.
91. The groups $(Z, +)$ and $(Q, +)$ are not isomorphic because
 (A) $(Z, +)$ is cyclic but $(Q, +)$ is not cyclic
 (B) Z is a proper subset of Q.
 (C) Not all the non zero elements in Z has multiplicative inverse in Z
 (D) $(Z, +)$ is a cyclic subgroup of $(Q, +)$.

92. $G = \left\{ \begin{pmatrix} x & x \\ x & x \end{pmatrix} : x \in \mathbb{R} \setminus \{0\} \right\}$ is a group under the multiplication. The inverse of $\begin{pmatrix} a & a \\ a & a \end{pmatrix}$ is

- (A) $\begin{pmatrix} \frac{1}{a} & \frac{1}{a} \\ \frac{1}{a} & \frac{1}{a} \end{pmatrix}$ (B) $\begin{pmatrix} -a & -a \\ -a & -a \end{pmatrix}$ (C) $\begin{pmatrix} a & a \\ a & a \end{pmatrix}$ (D) $\begin{pmatrix} \frac{1}{4a} & \frac{1}{4a} \\ \frac{1}{4a} & \frac{1}{4a} \end{pmatrix}$

93. The symmetric group S_n is

- (A) a non abelian group for any n (B) an abelian group for all n
 (C) non abelian only when $n \geq 3$ (D) abelian for $n \leq 3$.

94. The order of 2 and 3 respectively in (\mathbb{Z}_8, \oplus) is

- (A) 4 and 4 (B) 4 and 8 (C) 8 and 8 (D) 8 and 16.

95. The set of points $z \in \mathbb{C}$ for which $|z - 2| + |z + 2i| = 4$ is the conic

- (A) Hyperbola (B) Rectangle (C) Square (D) Ellipse

96. The set of points $z \in \mathbb{C}$ for which $|z + 2| - |z + 2i| = 4$ is the conic

- (A) Hyperbola (B) Rectangle (C) Square (D) Ellipse

97. If $f(z) = z|z|$, then $f(z)$ is differentiable

- (A) at all points z (B) only for $z=0$
 (C) at $z=1$ (D) nowhere

98. Let $n \in \mathbb{I}$. Then $e^{iz} = e^{\bar{iz}}$ is

- (A) For all z (B) only for $z = 2n\pi$
 (C) only for $z = n\pi$ (D) only for $z = (2n + 1)\pi$

99. The period of $\tan 2\pi z$ is

- (A) π (B) 2π (C) 2 (D) none

100. $\text{Log } z^n = n \text{Log } z$ is true for

- (A) None of the integers n (B) All z and positive integers n
 (C) Some z and some integers n (D) Such a relation is never possible