Examination: **M.Sc Mathematics**

SECTION 1 - SECTION 1

Question No.1

Let A be a square matrix of order 4 with $a_{ij} = 3$ for all i, j. Then the rank of A is

- 3
- 4
- 0
- 1

Question No.2

The radius of convergence of the series $\sum_{n=1}^{\infty} z^n / n^2$ is

- 1
- 5
- e
- 1/e

Question No.3

The coefficient of x^{2n} in the expansion of $\left(1+\frac{x^2}{2!}+\frac{x^4}{4!}+\ldots\right)^2$ is equal to

- $\frac{2^{2n}}{(2n)!}$
- $\frac{2^{2n}}{2(2n)!}$
- 0
- 1

Question No.4

In [0, 1] with usual metric, the closure of $A = Q \cap (0, 1)$ is

- (0, 1)
- A
- [0, 1)
- [0, 1]

Question No.5

Consider the group of all integers $\mathbb{Z}$ with respect to $*$ defined by $a*b = a+b+7$ for all $a, b \in \mathbb{Z}$. The the identity element in that group is

- −7
- 1
- −1
- 7

Question No.6

If $\vec{b}$ is a unit vector, then $(\vec{a}.\vec{b})\vec{b} + \vec{b} \times (\vec{a} \times \vec{b})$ is

- $|\vec{a}.\vec{b}|\vec{b}$
- $\vec{a}$
- $|\vec{a}.\vec{b}|\vec{a}$
- $\vec{b}$

Question No.7

Equation of the directrix of the parabola $y^2 + 4y+4x+2=0$ is

- x = 3/2
- x = -3/2
- x = -1
- x = 1

Question No.8

The value of
$\lim_{n\to\infty} ((1^2 + 2^2 + \ldots + n^2)/ n^2)$ is

- 1/6
- 1/3
- 1/4
- 1/2

Question No.9

Let A and B be two sets with
$|A| = 30$, $|B| = 45$ and $|A \cap B| = 10$.
Then $|A \Delta B|$ equals

- 45
- 55
- 65
- 85

Question No.10

The sum of the series $\sum_{n=0}^{\infty} \frac{n^2+n+1}{n!}$ is

$2e$

$4e$

$\frac{3}{2}e$

$3e$

Question No.11

Let f: R → R be defined by f(x) =1 if x is rational and f(x) =0 if x is irrational. Then f^{-1}([½, 3/2]) is

- R
- ϕ
- Q
- R - Q

Question No.12

$$\text{Let } f \text{ be defined by } f(x) = \begin{cases} \dfrac{1-\cos x}{x}, & x \neq 0; \\ k, & x = 0. \end{cases}$$

Then f is continuous at $x = 0$ if

- 0
- $-\frac{1}{2}$
- $\frac{1}{2}$
- $\frac{1}{4}$

Question No.13

$$\text{Expansion of the matrix } \begin{vmatrix} 1 & z & -y \\ -z & 1 & x \\ y & -x & 1 \end{vmatrix} \text{ gives}$$

- $1 + x + y + z$
- $1 + xyz$
- $1 + x^2 + y^2 + z^2$
-

$$x^2 + y^2 + z^2$$

Question No.14

Let G be a group of order 49. Then G is

- non-abelian
- non-cyclic
- abelian
- cyclic

Question No.15

Let H be a subgroup of a group G . Then____

- H is cyclic if G is nonabelian
- H is cyclic if G is cyclic
- H is cyclic if G is abelian
- H is cyclic

Question No.16

Two finite sets have m and n elements respectively. The total number of subsets of first set is 56 more than the total number of subsets of the second set. The values of m and n respectively are

- 6, 3
- 5, 1
- 8, 7
- 7, 6

Question No.17

The evolute of the cycloid
$x = a(\theta - \sin\theta)$ and $y = a(1 - \cos\theta)$ is

- another cycloid
- a straight line
- a circle
- acatenary

Question No.18

Which of the following subset of R is neither compact nor connected?

- (0, 1)
- Q
- R
- [0, 100]

Question No.19

If the standard deviation of the binomial distribution $(q+p)^{16}$ is 2, then mean of the distribution is

- 8
- 6
- 12
- 10

Question No.20

Let $f: R \to R$ be a continuous function and let $A = \{ x \in R \mid f(x) = 0 \}$. Then what best can you say of A?

- A is compact
- A is open
- A is closed
- A is bounded

Question No.21

If $y = e^{\tan x}$, then $\cos^2 x \frac{d^2y}{dx^2}$ is equal to

- $(1 - \sin 2x)\frac{dy}{dx}$
- $-(1 - \sin 2x)\frac{dy}{dx}$
- $-(1 + \sin 2x)\frac{dy}{dx}$
- $(1 + \sin 2x)\frac{dy}{dx}$

Question No.22

An equilateral triangle is inscribed in the circle $x^2 + y^2 = a^2$ with the vertex at (a,0). The equation of the side opposite to this vertex is

- 2x + a = 0
- 2x - a = 0
- x+a = 0
- 3x – 2a =0

Question No.23

The correct statement is _____

- $((-1)^n)$ is a convergent sequence
- $((-1)^n)$ is a bounded sequence
- $((-1)^n)$ is a divergent sequence
- $((-1)^n)$ is a monotonic sequence

Question No.24

The solution of $3x + 7y + 8z = -13; 2x + 9z = -5; -4x + y - 26z = 2$ is

- $x = 7, y = 0, z = -1$
- $x = -7, y = 0, z = 1$
- $x = 0, y = 0, z = 3$
- $x = -7, y = 0, z = 2$

Question No.25

In an ellipse , the distance between the focii is 6 and minor axis is 8. Then the eccentricity is

- 3/5
- 4/5
- 1 /√5
- 1/2

Question No.26

Let f: N → R be defined by f(n) = n/2 if n is even and f(n) = (1 - n)/ 2 if n is odd. The range of the sequence is

- N
- R
- Z
- Q

Question No.27

The focus of the parabola $4y^2 + 12x – 20y +67 = 0$ is

- (-3/4, 5/2)
- (-7/2, 5/2)
- (5/2, -3/4)
- (-17/4, 5/2)

Question No.28

Every square matrix satisfies its own characteristic equation. This is

- Cayley-Hamilton theorem
- Cauchy‟s theorem
- Eigen value theorem
- Sylow‟s theorem

Question No.29

In a discrete metric space , the only connected subsets are

- finite sets
- singleton sets

- the whole space
- all proper subsets

Question No.30

If $\lambda_1, \lambda_2, \ldots, \lambda_n$ are the eigen values of a matrix A of order n, then the eigen values of A^m are

- $\lambda_1^m, \lambda_2^m, \ldots, \lambda_n^m$
- $\frac{1}{m+\lambda_1}, \frac{1}{m+\lambda_2}, \ldots, \frac{1}{m+\lambda_n}$
- $m+\lambda_1, m+\lambda_2, \ldots, m+\lambda_n$
- $\frac{1}{\lambda_1^m}, \frac{1}{\lambda_2^m}, \ldots, \frac{1}{\lambda_n^m}$

Question No.31

In linear ordinary differential equation , the dependent variable and it differential coefficient are not multiple together and occurs only in ___________

- Fourth degree
- First degree
- Second degree
- Third degree

Question No.32

The solution of $\frac{dy}{dx} = \frac{y^2}{xy-x^2}$ is

- $e^{\frac{y}{x}} = ky$
- $e^{\frac{y}{x}} = kx$
- $e^{\frac{x}{y}} = kx$
- $e^{\frac{-y}{x}} = ky$

Question No.33

The number of non-trivial ideals in the ring of integers is

- 0
- 1
- finite
- infinite

Question No.34

The area lying in the first quadrant and bounded by the circle $x^2 + y^2 = 4$, the line $x = \sqrt{3}y$ and x -asis is

- π sq. unit
- $\frac{\pi}{2}$ sq. unit
- $\frac{\pi}{4}$ sq. unit
- $\frac{\pi}{3}$ sq. unit

Question No.35

Let A and B be two square matrices such that AB = A and BA = B. Then A and B are

- nilpotent
- idempotent
- periodic
- identity

Question No.36

Consider the sequence (a_n) = (1, 2, 3, 1, 2, 3, …) .Then lim sup an and lim inf a_n are respectively

- 3 and 2
- 1 and 3
- 3 and 1
- 2 and 3

Question No.37

The HCF and LCM of $x^2 + 3x$ and $3x + 9$ in $Z[x]$ is _________

- $x + 3,\ 3x^2 + 9x$
- $x^3 + 3x,\ x + 3$
- $9, 18$
- $x,\ x^2$

Question No.38

Let $f: R \to R$ be defined by $f(x) = x^2 + 1$. The values of $f^{-1}(\{17\})$ and $f^{-1}(\{-3\})$ are respectively,

- $\{4, -4\}$, ϕ
- ϕ , $\{3, -3\}$
- ϕ , $\{4, -4\}$

- $\{3, -3\}, \phi$

Question No.39

Every polynomial function of degree n > 1 has at least one zero. This is

- n-Root theorem
- Fundamental theorem of Algebra
- Cayley‟s theorem
- Lagrange‟s theorem

Question No.40

Let $\mathbb{R}$ be the set of all real numbers and $\mathbb{I} = \mathbb{R} \setminus \mathbb{Q}$ where $\mathbb{Q}$ be the set of all rational numbers. Then there exists

- an onto map from $\mathbb{Q}$ to $\mathbb{I}$
- a bijection from $\mathbb{Q}$ to $\mathbb{I}$
- infinitely many one-to-one map from $\mathbb{Q}$ to $\mathbb{I}$
- a one-to-one map from $\mathbb{Q}$ to $\mathbb{I}$

Question No.41

If g is function with the property that g{g(x)} = x + 1 for all real values of x and if g(0) = 1/2 , then g(1) is equal to

- 2
- $\frac{1}{2-x}$
- 1
- $\frac{1}{2}$

Question No.42

The series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\log(n+1)}$ is

- convergent
- absolutely convergent
- divergent
- oscillating

Question No.43

Let the rank of the matrix $\begin{bmatrix} -1 & 2 & 5 \\ 2 & -4 & a-4 \\ 1 & -2 & a+1 \end{bmatrix}$ is one, then the value of a is

- 2
- −1
- −6
- 1

Question No.44

Let f, g be two continuous functions defined on [0, 1] such that f(0) = g(1) =0, f(1) = g(0) = 1. Then there exists x ∈ (0, 1) such that f(x) = g(x) . This is due to

- Archemedian property
- Intermediate value property
- Heine-Borel property
- Neighbourhood property

Question No.45

$\cos\left(i\log\frac{a-ib}{a+ib}\right)$ is equal to

- $\frac{a^2-b^2}{a^2+b^2}$
- $\frac{2ab}{a^2+b^2}$
- $\frac{a^2-b^2}{2ab}$
- ab

Question No.46

Let f be a homomorphism from $(R,+) \to (R^*,\times)$. If $f(2) = 5$ then the value of $f(-8)$ is

- $\frac{1}{20}$
- 625

$\frac{1}{625}$

20

Question No.47

Sum of the focal distances of any point on the ellipse $x^2/a^2 + y^2/b^2 = 1$ is equal to the length of the the

- major axis
- minor axis
- latus rectum
- none of these

Question No.48

If $\frac{d}{dx}\{f(x)\} = \frac{1}{1+x^2}$, then $\frac{d}{dx}\{f(x^3)\}$ is

- $\frac{3x^2}{1+x^6}$
- $\frac{3x}{1+x^3}$
- $\frac{3}{2}$
- $\frac{-6x^5}{(1+x^6)}$

Question No.49

The pair of lines given by (a+2b)x+(a−3b)y = a−b for different values of a and b pass through the fixed point. The coordinates of the fixed point are

- $(\frac{2}{5}, \frac{2}{5})$
- $(\frac{2}{5}, \frac{3}{5})$
- $(\frac{3}{5}, \frac{3}{5})$
- $(\frac{1}{5}, \frac{1}{5})$

Question No.50

The number of solutions of tan x + sec x = 2 cos x in [0, 2 □] is

- 0
- 3
- 2

1

Question No.51

A real sequence can have

- finite number of limits
- at most one limit
- infinite number of limits
- at least one limit

Question No.52

Let V be the set if all polynomials of degree $\leq n$ in R[x]. A basis for V is

- $\{1, x, x^2, \ldots, x^{n-1}\}$
- $\{1, x, x^2, \ldots, x^{n+1}\}$
- $\{1, x, x^2, \ldots, x^n\}$
- $\{0, x, x^2, \ldots, x^n\}$

Question No.53

Let S be the set of integers. For $a, b \in S$, define a relation R by aRb if and only if $|a-b| < 1$. Then

- R is not symmetric
- R is not an equivalence relation
- $R = \{(a,a) : a \in S\}$
- R is not reflexive

Question No.54

The units of the ring Z[i] are

- ± 1
- $\pm i$
- $\pm 1, \pm i$
- ± 2

Question No.55

Let $T : \mathbb{R}^n \longrightarrow \mathbb{R}$ be a linear transformation. Then which of the following is a linear transformation on $\mathbb{R}^n$?

- $T_1 = T + 3$
- $T_1 = 5T$

- $T_1 = T^2$ where $T^2(x) = T(x).T(x)$ for all $x \in \mathbb{R}^n$
- $T_1 = 5T + 3$

Question No.56

The number of idempotent elements in any finite group is

- finite
- 2
- 1
- infinite

Question No.57

Let G = $Z_7 - \{0\}$ be the group under multiplication. The inverse of 3 is

- 3
- 4
- 2
- 5

Question No.58

The sum of n terms of the series, $\frac{1}{\sqrt{1}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{5}} + \frac{1}{\sqrt{5}+\sqrt{7}} + \ldots$ is

- $\frac{\sqrt{2n+1}}{2}$
- $\frac{\sqrt{2n+1}-1}{2}$
- $\sqrt{2n-1}$
- $\frac{1}{\sqrt{2n+1}}$

Question No.59

Let (1, 1) be an end of the diameter of a circle and the other end lies on the line x + y = 3. The locus of the centre of circle is

- 2(x − y) = 5
- 2(x + y) = 5
- 2x + y = 3
- x + y = 1

Question No.60

Let G be the Petersen graph. Which of the following is true?

- G is Eulerian

- G is Hamiltonian
- G is neither Eulerian nor Hamiltonian
- G is both Eulerian and Hamiltonian

Question No.61

If the median of 21 observations is 40 and if the observations greater than the median are increased by 6, then the median of the new data will be

- 86
- 46
- 40
- 61

Question No.62

If S is an ideal of a ring R that contains a unit of R, then what best can you say of S?

- $S \subset R$
- $S = \phi$
- $S \subseteq R$
- $S = R$

Question No.63

For the curve $r = a(1 + \cos\theta)$, the tangent at the pole is

- $\theta = \pi$
- $\theta = \pi/4$
- $\theta = 0$
- $\theta = \pi/2$

Question No.64

The degree of each vertex of a certain graph of order 12 and size 31 is either 4 or 6. How many vertices of degree 4 are there?

- 3
- 9
- 5
- 7

Question No.65

Let A = { 0, 1, ½, 1/3, …, 1/n, .. .}. Then the derived set of A is

- {1}
- {0}
- {0, 1}
- A

Question No.66

The sum and product of the eigen values of the 3 X 3 identity matrix are respectively

- 3, 1
- 2, 1

- 3, 3
- 0, 0

Question No.67

Two tangents are drawn from the origin to a circle with centre at (2, -1). If the equation of one of the tangents is 3x+y = 0, the equation of the other tangent is

- x + 3y = 0
- x – 3y = 0
- x + 2y = 0
- 3x – y = 0

Question No.68

Let f: R → R and g: R → R be uniformly continuous functions. Which of the following is not true?

- 2f + 3g is uniformly continuous
- f + g is uniformly continuous
- fg is uniformly continuous
- f - g is uniformly continuous

Question No.69

Let $f(2) = 2$ and $f'(2) = 1$. Then $\lim_{x\to 2} \dfrac{2x^2 - 4f(x)}{x-2}$ is equal to

- −4
- 2
- −2
- 4

Question No.70

Which of the following series is not convergent?

- $\sum 1/n^3$
- $\sum 1/n$
- $\sum 1/n!$
- $\sum 1/n^4$

Question No.71

Let z be a complex number satisfying $\left|\dfrac{z-25}{z-1}\right| = 5$. Then the value of $|z|$ is

- 3
- 4
- 6
- 5

Question No.72

If $a_n = \frac{2^n n!}{n^n}$ then $\lim_{n\to\infty} \frac{a_n}{a_{n+1}} =$

- 2e
- e
- $\frac{e}{2}$
- $\frac{1}{e}$

Question No.73

The equation $\frac{x^2}{2-\lambda} - \frac{y^2}{\lambda-5} - 1 = 0$ represents an ellipse if

- 2 > λ > 5
- λ < 2
- λ > 5
- 2 < λ < 5

Question No.74

The kernel of the homomorphism
$f : (Z, +) \to (R^*, .)$ defined by $f(x) = 2^x$ is

- {0}
- {1, -1}
- {1}
- Z

Question No.75

The number of generators of the group $(Z_{12}, \oplus)$ is

- 3
- 4
- 1
- 2

Question No.76

Which of the following is a compact subset of R with usual metric?

- R
- {1, ½, 1/3, …, 1/n, …}
- [0, 5]

(0, 10)

Question No.77

The number of edges in the complete bipartite graph $K_{m,n}$ is

- m
- n
- mn
- m+n

Question No.78

Four persons are selected at random from a group of 3 men, 2 women and 4 children. The probability that exactly two of them are children is

- 10/23
- 1/2
- 10/21
- 9/21

Question No.79

If a real valued function f is continuous on $[a,b]$ and differential on (a,b) and $f'(x) \neq 0$ in (a,b) then

- f is strictly monotonic in $[a,b]$
- f is monotonic in (a,b)
- $f(x)$is constant on $[a,b]$
- $f$is monotonic in $[a,b]$

Question No.80

Applying the Cauchy's root test the series $\sum_{n=1}^{\infty}\left(\frac{n}{2n+1}\right)^{n}$ is

- Convergent
- neither convergent nor divergent
- divergent
- both convergent and divergent

Question No.81

Let G be a group of order 2p where p is a prime. Let H be a normal subgroup of order p. Then the index of H in G is

- p
- 1
- 3
- 2

Question No.82

Subset of R which is a neighborhood of 3 is

- $(3,6)$
- $(2,4)$
- $[3,6]$
- $[3,6)$

Question No.83

Let G be the group of non-zero complex numbers under multiplication. The order of –i is

- 2
- 1
- ∞
- 4

Question No.84

$$\text{Let } a_n = \begin{cases} \frac{1}{n^2} & n \text{ is even;} \\ \frac{1}{n} & n \text{ is odd.} \end{cases}$$

Then the series $\sum_{n=1}^{\infty} a_n$ is

- oscillating
- uniformly convergent
- divergent
- convergent but not uniform

Question No.85

Let (M, d) be any metric space and let A, B be subsets of M. Which of the following is not true?

- $\mathrm{Int}(A \cup B) = \mathrm{Int}(A) \cup \mathrm{Int}(B)$
- A is open iff $A = \mathrm{Int}\, A$
- $\mathrm{Int}(A \cap B) = \mathrm{Int}(A) \cap \mathrm{Int}(B)$
- $A \subseteq B$ implies $\mathrm{Int} A \subseteq \mathrm{Int}\, B$

Question No.86

The number of peak points of the sequence (1, 2, 3, …) is

- 3
- 2
- 1
- 0

Question No.87

If $y = \sqrt{x \log_e x}$, then $\dfrac{dy}{dx}$ at $x = e$ is

- $\frac{1}{\sqrt{e}}$
- $-\frac{1}{\sqrt{e}}$
- $\frac{1}{e}$
- $-\frac{1}{e}$

Question No.88

Let $\{U_n\}$ and $\{V_n\}$ are two bounded sequences, then $\{U_n + iV_n\}$ is

- Oscillation sequence
- Constant sequence
- Un bounded sequence
- Bounded sequence

Question No.89

The period of the function f(x) = $\sin^{-1}(\sin x) + e^{\tan x}$ is

- 2π
- $\frac{\pi}{2}$
- $\frac{\pi}{4}$
- π

Question No.90

Let M be the metric space of the set of integers with discrete metric. The diameter of A = { 1, 5, 7, 9} is

- 1
- 0
- 9
- 8

Question No.91

The general solution of the equation is ydx-xdy = 0 is ______

- $\dfrac{x}{y} = c$
-

x+y =c

x-y=c

xy=c

Question No.92

Let $f(x) = \frac{x-1}{4} + \frac{(x-1)^3}{12} + \frac{(x-1)^5}{20} + \frac{(x-1)^7}{28} + \ldots,$ where $0 < x < 2$. Then $f'(x)$ is equal to

- $\frac{1}{4x(2-x)}$
- $\frac{1}{2-x}$
- $\frac{1}{2+x}$
- $\frac{1}{4(x-2)^2}$

Question No.93

The function f(x) = sin x+ cos x is

- an odd function
- an even function
- neither odd nor even
- both odd and even

Question No.94

Which one of the following is not a subspace of $\mathbb{R}^3$?

- $\{(a, a+b, -a+2b) : a, b \in \mathbb{R}\}$
- $\{(a, b, c) : a, b, c, \in \mathbb{Q}\}$
- $(0, 0, 0)$
- $\{(a, a-b, b) : a, b \in \mathbb{R}\}$

Question No.95

Let G be a cyclic group of order 60. Then the number of non-trivial subgroups of G is

- 4
- 3
- 1
- 10

Question No.96

Let Z be the set of integers. The function $f: Z \rightarrow Z$ defined by $f(x) = 3x$ is

- 1-1 but not onto
- neither 1-1 nor onto
- onto but not 1-1
- 1-1 and onto

Question No.97

In the expansion of $\left(x + \frac{a}{x^2}\right)^n$ $(a \neq 0)$, if term independent of x does not exist, then n must be

- 15
- 18
- 12
- 16

Question No.98

The zeros of the function $(z+1)^2 (iz+2)^3 / (z+7)$ are

- 1 and -2/i
- -1 and -2/i
- 0, 1 and 2/i
- -1 and 2/i

Question No.99

The inequality $|x - 2| < 6$ can be expressed in the form $a < x < b$ where a = _____and b = ____.

- 4,9
- 5,6
- -4, 8
- 2,3

Question No.100

The value of $\iint dx\, dy$ over the region bounded by x=0, x=2, y=0, y=2 is

- 2
- 0
- 4
- 3