ENTRANCE EXAMINATION FOR ADMISSION, MAY 2010.

M.Sc. FIVE YEAR INTEGRATED PROGRAMME (MATHEMATICS, COMPUTER SCIENCE AND STATISTICS)

COURSE CODE : 384

Register Number :

Signature of the Invigilator
(with date)

COURSE CODE : 384

Time : 2 Hours

Max : 400 Marks

Instructions to Candidates :

1. Write your Register Number within the box provided on the top of this page and fill in the page 1 of the answer sheet using pen.
2. Do not write your name anywhere in this booklet or answer sheet. Violation of this entails disqualification.
3. Read each question carefully and shade the relevant answer (A) or (B) or (C) or (D) in the relevant box of the ANSWER SHEET using HB pencil.
4. Avoid blind guessing. A wrong answer will fetch you –1 mark and the correct answer will fetch 4 marks.
5. Do not write anything in the question paper. Use the white sheets attached at the end for rough works.
6. Do not open the question paper until the start signal is given.
7. Do not attempt to answer after stop signal is given. Any such attempt will disqualify your candidature.
8. On stop signal, keep the question paper and the answer sheet on your table and wait for the invigilator to collect them.
9. Use of Calculators, Tables, etc. are prohibited.

1. The number of terms are there in 2,4,8,16, ———— 1024 is

(A) 512 (B) 20 (C) 10 (D) 17

2. The unit's digit in the product $(7^{71} \times 6^{59} \times 3^{65})$ is

(A) 1 (B) 2 (C) 4 (D) 7

3. The difference between the squares of two consecutive odd integers is always divisible by

(A) 8 (B) 6 (C) 3 (D) 11

4. The H.C.F of 1.75, 5.6 and 7 is

(A) 3.5 (B) 0.35

(C) 0.75 (D) None of the above

5. Two pens and three pencils cost Rs. 86. Four pens and a pencil cost Rs. 112. The cost of a pen and that of a pencil is

(A) Cost of pen = Rs. 25 and cost of a pencil = Rs. 12

(B) Cost of pen = Rs. 12 and cost of a pencil = Rs. 25

(C) Cost of pen = Rs. 16 and cost of a pencil = Rs. 22

(D) None of the above

6. The square root of 1471369 is

(A) 2123 (B) 2313 (C) 1213 (D) 1516

7. The value of

$$\sqrt{10+\sqrt{25+\sqrt{108+\sqrt{154+\sqrt{225}}}}}$$

(A) 4 (B) 6 (C) 8 (D) 9

8. The value of

$$\sqrt{.0025} \times \sqrt{2.25} \times \sqrt{.0001} = ?$$

(A) .000075 (B) .0075

(C) .075 (D) None of the above

9. Find the average of first 40 natural numbers.

(A) 19.5 (B) 20 (C) 20.5 (D) 25.7

10. If $A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -1 \\ 3 & -1 & 1 \end{bmatrix}$ then

(A) $A^3 + 3A^2 + A - 9I_3 = 0$ (B) $A^3 - 3A^2 + A + 9I_3 = 0$

(C) $A^3 - 3A^2 - A + 9I_3 = 0$ (D) $A^3 - 7A^2 - A + 9I_3 = 0$

11. $\lim_{x \to \infty} \frac{\sin x}{x} = ?$

(A) 0 (B) 1

(C) ∞ (D) No limit exist

12. $\lim_{x \to 0} \frac{a^x - b^x}{x} = ?$

(A) 0 (B) 1 (C) log(a/b) (D) log(ab)

13. $\lim_{x \to 0} (1 + x)^{\frac{1}{x}} = ?$

(A) e^2 (B) 1/e

(C) $1/e^2$ (D) None of the above

14. The distance between the lines $4x + 3y = 9$ and $8x + 6y = 15$ is

(A) 3 (B) 0.3 (C) 0.6 (D) 0.7

15. A polygon has 54 diagonals. Number of sides of this polygon is

(A) 12 (B) 27 (C) 16 (D) 19

16. If $\log x + \log y = \log(x + y)$, then

(A) $x = y$ (B) $xy = 1$

(C) $y = x - 1/x$ (D) None of the above

17. If $\log 2 = 0.30103$, the number of digits in 4^{50} is

(A) 30 (B) 31 (C) 32 (D) 71

18. The number of iron rods, each of length 7 m and diameter 2 cm can be made out of 0.88 cubic metre of iron is

(A) 100 (B) 200 (C) 400 (D) 550

19. The ratio of the diameters of two spheres is 4:5. Then the ratio of their surface areas is

(A) 16 : 25 (B) 9 : 1

(C) 3 : 1 (D) None of the above

20. The number of bricks, each measuring 24 cm × 12 cm × 8 cm, required to Construct a wall 24 m long, 8 m high and 60 cm thick, if 10% of the wall is filled with mortar is

(A) 35000 (B) 40000 (C) 45000 (D) 6000

21. A man and his wife appear in an interview for two vacancies in the same post. The probability of husband's selection is 1/7 and the probability of wife's selection is 1/5. What is the probability that only one of them is selected is

(A) 4/5 (B) 2/7 (C) 8/15 (D) 9/25

22. A ladder leaning against a wall makes an angle of 60 degree with the ground. If the length of the ladder is 19m, then the distance of the foot of the ladder from the wall is

(A) 9 m (B) 9.5 m (C) 10 m (D) 15 cm

23. If three solid cubes of sides 1cm, 6cm and 8cm are melted to form a new cube then the surface area of the cube so formed is given by

(A) 4860 cm^2 (B) 468 cm^2 (C) 486 cm^2 (D) 525 cm^2

24. If one seventh of a number exceeds its eleventh part by 100 then the number is,

(A) 770 (B) 1100 (C) 1925 (D) 2660

25. If a/b = 4/3, then the value of 6a + 4b/6a – 4b is

(A) –1 (B) 3 (C) 4 (D) 7

26. If $3x + 7 = x^2 + p = 7x + 5$ the value of p is

(A) 1/2 (B) 8 ¼ (C) 8 ½ (D) None of these

27. The simplest value of

$$\frac{\sqrt{5}+2}{\sqrt{5}-2}+\frac{\sqrt{5}-2}{\sqrt{5}+2}$$

(A) 9 (B) 18 (C) 14 (D) 36

28. A continuous random variable X has the following pdf $f(x) = \{kx(1-x)^{10}, 0< x< 1$, elsewhere $f(x) = 0$, then the value of k is

(A) 130 (B) 134 (C) 133 (D) 132

29. If X is a continuous random variable, the function given by $F(x)=P(X \leq x)$ is known as

(A) Probability density function (B) Probability mass function

(C) Cumulative distribution function (D) Survival function

30. If a random variable takes only a finite or a countable number of values, it is called as

(A) Discrete random variable

(B) Continuous random variable

(C) Stochastic variable

(D) Discrete continuous random variable

31. Let X be the random variable denoting the amount that a person can win. The possible values of X with its corresponding probability values are tabulated below

X	20	40	–30	0
P(x)	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{2}$

Then E(X) is equal to

(A) Rs. 5 (B) Rs. 15 (C) Rs. 25 (D) Rs. 35

32. The mean and variance of the random variable X having pdf $f(x)= 3\ e^{-3x}$, $0 < x < \infty$, elsewhere $f(x) = 0$ is

(A) Mean = 3, Variance = 9

(B) Mean = 2, Variance = 8

(C) Mean = $\frac{1}{2}$, Variance = $\frac{1}{8}$

(D) Mean = $\frac{1}{3}$, Variance = $\frac{1}{9}$

33. The difference between the mean and the variance of a Binomial distribution is 1 and the difference between their squares is 11. Then the value of n is

(A) 20 (B) 36 (C) 13 (D) 35

34. A random variable X is said to have a Poisson distribution with the probability mass function $P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!}$, then the range and mean value of X is

(A) x=1,2,3,3...n; λ^2

(B) x=0,1,2,3,...; λ

(C) x=0,1,2,3,....n; λ^3

(D) x=0,1,2,3,....; λ^2

35. In a Poisson distribution if P(X = 2) = P(X = 3) then the value of P(X = 5) is (given e^{-3} =0.050)

(A) 0.101 (B) 0.201 (C) 0.110 (D) 0.120

36. The number of telephone calls received at a telephone exchange in a given time interval is an example of

(A) Binomial distribution

(B) Poisson distribution

(C) Normal distribution

(D) Exponential distribution

37. If f(x) is a pdf of a Normal variate X with mean μ and variance σ^2 then $\int_0^\infty f(x)dx$ is equal to

(A) 0 (B) 1 (C) 0.5 (D) –0.5

38. If 2 cards are drawn from a well shuffled pack of 52 cards, the probability that they are of the same colours is

(A) $\frac{1}{2}$ (B) $\frac{26}{51}$ (C) $\frac{25}{51}$ (D) $\frac{25}{102}$

39. A box contains 6 red and 4 white balls. If 3 balls are drawn at random, the probability of getting 2 white balls is

(A) $\frac{1}{20}$ (B) $\frac{18}{125}$ (C) $\frac{4}{25}$ (D) $\frac{3}{10}$

40. The value of V(4X + 3), if V(X) = 2 is

(A) 35 (B) 30 (C) 12 (D) 11

41. In 5 throws of a die, getting 1 or 2 is a success. The mean number of successes is

(A) $\frac{5}{3}$ (B) $\frac{3}{5}$ (C) $\frac{5}{9}$ (D) $\frac{9}{5}$

42. If μ_2 =20 , μ_2' = 276 for a discrete random variable X, then the mean of the random variable X is

(A) 16 (B) 5 (C) 2 (D) 1

43. A batsman has a certain average of runs for 16 innings. In the 17th innings, he makes a score of 85 runs, thereby increasing his average by 3. Then the average after the 17th innings is

(A) 37 (B) 36 (C) 35 (D) 34

44. The average of 13 numbers is 68 , the average of first seven numbers is 63 and the average of last seven numbers is 70. Then the 7th number is

(A) 46 (B) 47 (C) 48 (D) 50

45. In a mixture of 35 litres, the ratio of milk and water is 4 : 1. If one litre of water is added to mixture, then what will be the new ratio of milk and water?

(A) 7 : 3 (B) 7 : 4 (C) 7 : 2 (D) 7 : 1

46. A committee of 3 members is to be selected out of 3 men and 2 women. What is the probability that the committee has at least one women?

(A) $\frac{1}{10}$ (B) $\frac{9}{20}$ (C) $\frac{1}{20}$ (D) $\frac{9}{10}$

47. Two numbers are in the ratio 2:3. 20% of the smaller number added to 20 becomes equal to the sum of 10% of the larger number and 25. Then the smallest number is

(A) 100 (B) 150 (C) 50 (D) 200

48. When 75% of a number is added to 75, the result is the number again. Then the number is

(A) 250 (B) 300 (C) 400 (D) 450

49. If A and B are two sets such that n(A) = 20, n(A $\cup$ B) =30, n(A $\cap$ B)= 5 then n(B) is equal to

(A) 12 (B) 10 (C) 15 (D) 20

50. If $A + B = 180$, then the value of $\sin^2 A - \sin^2 B$ is equal to

(A) –1 (B) 0 (C) 1 (D) $\sqrt{2}$

51. The equation whose roots are the squares of the roots of $x^2 - 3x + 2 = 0$ is

(A) $x^2 - 5x + 4 = 0$ (B) $x^2 + 2x + 6 = 9$

(C) $x^2 + 6x + 8 = 14$ (D) $x^2 + 4x - 5 = 0$

52. If $x + \frac{1}{x} = 4$ then the value of $x^4 + \frac{1}{x^4}$ is equal to

(A) 109 (B) 125 (C) 112 (D) 194

53. The three sides of a triangles are 3 cm, 4 cm and 5 cm. Then the area of the triangle is

(A) 6 square units (B) 5 square units

(C) 10 square units (D) 8 square units

54. The radius of a wheel is 21 cm. How many revolution will it make in traveling 924 meters? (use $\pi = 22/7$)

(A) 7 (B) 11 (C) 200 (D) 700

55. Out of 100 students, 50 failed in English and 30 failed in Mathematics. If 12 students failed in both English and Mathematics, then the number of students passed in both the subjects is

(A) 34 (B) 33 (C) 32 (D) 30

56. There are 30 boys in a class and the average age is 15.1 years. Three new boys are put in this class, which made the average age 15.2 years. One new boy is aged 16 years, the other two are twins. Then the age of the twins are

(A) 14 (B) 16.3 (C) 17.2 (D) 18

57. The 16th term of the series 1,3,6,10,15,..... is equal to

(A) 132 (B) 136 (C) 126 (D) 120

58. A point on the parabola $y^2 = 2x$ that is closest to that point (1,4) is

(A) (2, 2) (B) (3, 3) (C) (2, 3) (D) (3, 2)

59. The area between the curves $y = x^2 - x - 2$, x-axis and the lines $x = -2$ and $x = 4$ is

(A) 10 square units (B) 15 square units

(C) 17 square units (D) 21 square units

60. The equation of the curve whose slope at any point is equal to $y + 2x$ and which passes through the origin is

(A) $y = \sin x$ (B) $y = e^x$

(C) $y = 2(e^x - x - 1)$ (D) $y = x^2 - x + 1$

61. A cup of coffee at temperature 100°C is placed in a room whose temperature is 15°C and it cools to 60°C in 5 minutes. Its temperature after a future interval of 5 minutes is

(A) 38°C (B) 38.82°C (C) 32°C (D) 27.5°C

62. The most appropriate solution to the equations

$x+y+2z=0$; $3x+2y+z=0$; $2x+y-z=0$ is

(A) (3, –5, 1) (B) (3k, -5k, k), k is any integer

(C) (0, 0, 0) (D) None of the above

63. The work done by the force $\vec{F} = a\vec{i} + \vec{j} + \vec{k}$ in moving the point of application from (1, 1, 1) to (2, 2, 2) along a straight line is given to be 5 units, where the value of 'a' is

(A) 3 (B) 6 (C) 9 (D) 12

64. The angle between the lines A = $\vec{i} - 2\vec{j} - 2\vec{k}$ and B = $6\vec{i} + 3\vec{j} + 2\vec{k}$ is

(A) $\cos^{-1}\left(\frac{19}{21}\right)$ (B) $\cos^{-1}\left(\frac{20}{21}\right)$ (C) $\cos^{-1}\left(\frac{-4}{21}\right)$ (D) $\cos^{-1}\left(\frac{20}{\sqrt{21}}\right)$

65. The distance between the parallel planes $\vec{r}.(-\vec{i} - \vec{j} + \vec{k}) = 3$ and $\vec{r}.(\vec{i} + \vec{j} - \vec{k}) = 5$ is

(A) $\frac{8}{\sqrt{3}}$ (B) 3 (C) 8 (D) $\frac{5}{\sqrt{3}}$

66. $(1+i)^n + (1-i)^n$ is equal to

(A) $2^{\frac{n+2}{2}} \cos \frac{n\pi}{4}$ (B) $2^{\frac{n+2}{2}} \sin \frac{n\pi}{4}$

(C) $2^{\frac{n}{2}} \cos \frac{n\pi}{4}$ (D) $2 \cos n\pi$

67. If a parabolic reflector is 20 cm in diameter and 5 cm deep, then the distance of the focus from the centre of the reflector is

(A) 7 cm (B) 5 cm (C) 3 cm (D) 10 cm

68. The point inflection of the curve is $y = x^4$ is at

(A) $x = 0$ (B) $x = 3$ (C) $x = 12$ (D) nowhere

69. In which region the curve $y^2(a+x) = x^2(3a-x)$ does not lie?

(A) $x > 0$ (B) $0 < x < 3a$

(C) $x \leq -a$ and $x > 3a$ (D) $-a < x < 3a$

70. The complementary function of $(D^2+1)y = e^{2x}$ is

(A) $(Ax + B)\, e^x$ (B) $A \cos x + B \sin x$

(C) $(Ax + B)\, e^{2x}$ (D) $(Ax + B)\, e^{-x}$

71. The differential equation satisfied by all the straight lines in xy plane is

(A) $\frac{dy}{dx}$ = a constant (B) $\frac{d^2y}{dx^2} = 0$

(C) $y + \frac{dy}{dx} = 0$ (D) $\frac{d^2y}{dx^2} + y = 0$

72. Which of the following are statements?

(i) May God bless you (ii) Rose is a flower

(iii) Milk is white (iv) 1 is a prime number

(A) (i),(ii),(iii) (B) (i), (ii), (iv)

(C) (i),(iii),(iv) (D) (ii), (iii), (iv)

73. Which of the following is a tautology?

(A) $p \vee q$ (B) $p \wedge q$ (C) $p \vee \sim q$ (D) $p \wedge \sim q$

74. If $\vec{a}$ and $\vec{b}$ are two unit vectors and θ is the angle between them, then $(\vec{a} + \vec{b})$ is a unit vector if

(A) $\theta = \frac{\pi}{3}$ (B) $\theta = \frac{\pi}{4}$ (C) $\theta = \frac{\pi}{2}$ (D) $\theta = \frac{2\pi}{3}$

75. If - $\overline{Z}$ lies in the third quadrant then Z lies in the

(A) first quadrant (B) second quadrant

(C) third quadrant (D) fourth quadrant

76. The polar form of the complex number $(i^{25})^3$ is

(A) $\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}$ (B) $\cos\pi+i\sin\pi$

(C) $\cos\pi-i\sin\pi$ (D) $\cos\frac{\pi}{2}-i\sin\frac{\pi}{2}$

77. The tangents at the end of any focal chord to the parabola $y^2=12x$ intersect on the line

(A) $x-3=0$ (B) $x+3=0$ (C) $y+3=0$ (D) $y-3=0$

78. If the projection of $\vec{a}$ on $\vec{b}$ and projection of $\vec{b}$ on $\vec{a}$ are equal then the angle between $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ is

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{4}$ (D) $\frac{2\pi}{3}$

79. The modules and amplititude of the complex number $\left[e^{3-i\frac{\pi}{4}}\right]^3$ are respectively

(A) $e^9, \frac{\pi}{2}$ (B) $e^9, \frac{-\pi}{2}$ (C) $e^6, \frac{-3\pi}{4}$ (D) $e^9, \frac{-3\pi}{4}$

80. The diretrix of the parabola $y^2 = x+4$ is

(A) $x=\frac{15}{4}$ (B) $x=-\frac{15}{4}$ (C) $x=-\frac{17}{4}$ (D) $x=\frac{17}{4}$

81. The value of $i+i^{22}+i^{23}+i^{24}+i^{25}$ is

(A) i (B) $-i$ (C) 1 (D) –1

82. The eccentricity of the hyperbola with asymptotes $x+2y-5=0,\ 2x-y+5=0$ is

(A) 3 (B) $\sqrt{2}$ (C) $\sqrt{3}$ (D) 2

83. The values of t for which $\begin{vmatrix} t-2 & 3 \\ 4 & t-1 \end{vmatrix} = 0$ are

(A) 4, 0 (B) 6, 2 (C) –5, 2 (D) 5, –2

84. A square matrix A of order n with $|A| = 0$, the rank of the matrix A is

(A) Less than n (B) Equal to n

(C) More than n (D) Unity

85. The value of $\log_5 5.\log_2 9.\log_3 2$ is

(A) 2 (B) 1 (C) 5 (D) 4

86. The value of $\int_0^1 (x^2 - 3x + 5)dx$ is

(A) $\frac{23}{3}$ (B) $\frac{23}{4}$ (C) $\frac{23}{5}$ (D) $\frac{23}{6}$

87. $y = f(x)$ is twice differentiable and has a minimum value, then

(A) $f'(c) = 0$ *and* $f''(c) < 0$ (B) $f'(c) = 0$ *and* $f''(x) > 0$

(C) $f'(c) = 0$ *and* $f''(c) = 0$ (D) None of the above

88. The slope of the equation 5y–3x = 6 is

(A) $\frac{1}{5}$ (B) $\frac{2}{5}$ (C) $\frac{3}{5}$ (D) $\frac{4}{5}$

89. The value of 'C' of Lagranges Mean Value Theorem for f(x) = $\sqrt{x}$ when a = 1 and b = 4 is

(A) $\frac{9}{4}$ (B) $\frac{3}{2}$ (C) $\frac{1}{2}$ (D) $\frac{1}{4}$

90. Which of the following is increasing in (0, ∞) ?

(A) e^x (B) $\frac{1}{x}$ (C) $-x^2$ (D) x^{-2}

91. The function y = tan x – x is

(A) an increasing function in $(0, \frac{\pi}{2})$

(B) a decreasing function in $(0, \frac{\pi}{2})$

(C) increasing in $(0, \frac{\pi}{4})$ and decreasing in $(\frac{\pi}{4}, \frac{\pi}{2})$

(D) decreasing in $(0, \frac{\pi}{4})$ and increasing in $(\frac{\pi}{4}, \frac{\pi}{2})$

92. Which of the following curves in concave down?

(A) $y = -x^2$ (B) $y = x^2$

(C) $y = e^x$ (D) $y = x^2 + 2x - 3$

93. If $u = x^y$ then $\frac{\partial y}{\partial x}$ is equal to

(A) y x^{y-1} (B) u log x (C) u log y (D) x y^{x-1}

94. If u = $\left(\frac{x^4 + y^4}{x^2 + y^2}\right)$ and f = sin u then f is a homogeneous function of degree

(A) 0 (B) 1 (C) 2 (D) 4

95. The curve $a^2y^2 = x^2(a^2 - x^2)$ has

(A) only one loop between $x = 0$ and $x = a$

(B) two loops between $x = 0$ and $x = a$

(C) two loops between $x = -a$ and $x = a$

(D) no loop

96. The volume of the solid obtained by revolving $\frac{x^2}{9} + \frac{y^2}{16} = 1$ about the minor axis is

(A) 48 π (B) 64 π (C) 32 π (D) 128 π

97. Integrating factor of $\frac{dy}{dx} + \frac{1}{x \log x}.y = \frac{2}{x^2}$ is

(A) e^x (B) log x (C) $\frac{1}{x}$ (D) e^{-x}

98. Which of the following is not a group?

(A) $(Z_n, +_n)$ (B) (Z, +) (C) (Z, .) (D) (R, +)

99. The order of $[7]$ in $(Z_9, +_9)$ is

(A) 9 (B) 6 (C) 3 (D) 1

100. In the multiplicative group of nth roots of unity, the inverse of ω^k is (k < n) is

(A) $\omega^{\frac{1}{k}}$ (B) ω^{-1} (C) ω^{n-k} (D) $\omega^{\frac{n}{k}}$