21/14

ENTRANCE EXAMINATION FOR ADMISSION, MAY 2010.

M.Sc. (MATHEMATICS)

COURSE CODE : 372

Register Number :

Signature of the Invigilator (with date)

COURSE CODE : 372

Time : 2 Hours

Max : 400 Marks

Instructions to Candidates :

1. Write your Register Number within the box provided on the top of this page and fill in the page 1 of the answer sheet using pen.
2. Do not write your name anywhere in this booklet or answer sheet. Violation of this entails disqualification.
3. Read each question carefully and shade the relevant answer (A) or (B) or (C) or (D) in the relevant box of the ANSWER SHEET using HB pencil.
4. Avoid blind guessing. A wrong answer will fetch you –1 mark and the correct answer will fetch 4 marks.
5. Do not write anything in the question paper. Use the white sheets attached at the end for rough works.
6. Do not open the question paper until the start signal is given.
7. Do not attempt to answer after stop signal is given. Any such attempt will disqualify your candidature.
8. On stop signal, keep the question paper and the answer sheet on your table and wait for the invigilator to collect them.
9. Use of Calculators, Tables, etc. are prohibited.

1. If $f(x) = \begin{cases} -x \text{ for } -1 \leq x \leq 0 \text{ and} \\ x^2 \text{ for } 0 < x \leq 1 \end{cases}$

then

(A) f is continuous but not differentiable at 0

(B) f is not continuous but differentiable at 0

(C) f is differentiable everywhere

(D) none of these

2. If F is a closed subset of R containing all the rational numbers then

(A) F = R

(B) F is compact

(C) F = Q

(D) None of these

3. $\lim_n \frac{2^n}{n!}$ is

(A) 0

(B) ∞

(C) 2

(D) not existing

4. The set $\{e^{2+iy} : y \text{ belongs to } R\}$ is

(A) compact

(B) not bounded

(C) not closed

(D) finite

5. If $W = \{(x_1, x_2, \ldots, x_n) \text{ belongs to } R^n : \sum_1^n x_i = 0\}$ where $n \geq 2$ then dimension of W is

(A) n

(B) n – 1

(C) 0

(D) ∞

6. $\lim_n \inf \left[(-1)^n \left(1 + \frac{1}{n}\right)\right]$ is

(A) 1

(B) 0

(C) – 1

(D) ∞

7. Suppose $n \geq 1$. If V is the set of all polynomials of degree $\leq n$ with integer coefficients then it is not a vector space over R since

(A) it is not closed under addition

(B) it is closed under addition, but (V, +) does not form an abelian group

(C) it is not closed under scalar multiplication

(D) 0 does not belong to V

8. The function $e^{x} - x - 1$ is strictly increasing on

(A) $(0, \infty)$ (B) nowhere

(C) $(-\infty, 0)$ (D) $(-\infty, \infty)$

9. The number of generators of the group (Z, +) is

(A) 2 (B) 1 (C) 0 (D) ∞

10. The closure of $(0, \infty)$ in R is

(A) $(0, \infty)$ (B) $(0, \infty]$ (C) $[0, \infty]$ (D) $[0, \infty)$

11. If $f(x) = \lim_{n \to \infty} (\sin x)^{2n}$, then f is

(A) continuous at $x = \frac{\pi}{2}$

(B) discontinuous at $x = (2k + 1)\frac{\pi}{2}$, k in Z

(C) continuous at $x = -\frac{\pi}{2}$

(D) f is continuous at infinite number of points

12. The center of an abelian group is

(A) equal to the identity (B) equal to the whole group

(C) always nontrivial and normal (D) not normal

13. $f(z) = |z|^2$ is

(A) analytic on C (B) not continuous

(C) differentiable at 0 and nowhere else (D) differentiable nowhere in C

14. If x is positive then the set of all y for which the relation $(1 + x)^{y} > 1 + xy$ holds is equal to

(A) Z (B) N (C) Q (D) R+

15. If $a_n = \frac{(-1)^n n}{n+1}$ then lim sup a_n is equal to

(A) 0 (B) 1 (C) ∞ (D) does not exist

16. Let f : R → R be a function such that f(0) > 0. Then there exists a $\delta > 0$ such that f(x) > 0 for all x belonging to $(-\delta, \delta)$

(A) always

(B) only for continuous functions

(C) only for differential functions

(D) only for monotone continuous functions

17. Under addition and multiplication modulo p, where p is a prime number, the set {0, 1, 2, ..., p–1}is

(A) a commutative ring but not a field (B) a non-commutative ring

(C) a field (D) a skew field

18. If $x_r + \frac{1}{x_r} = 2.\cos\frac{\pi}{2^r}$, *then* $x_1.x_2.x_3.........\infty$ *is*

(A) 1 (B) –1

(C) 0 (D) indeterminate

19. In a group of 100 people, 75 can speak English, 87 Hindi, 90 Bengali and 95 Urdu. The number of people who can speak all the 4 languages is

(A) 0 (B) 37 (C) 47 (D) 55

20. Let ω be the imaginary cube root of unity. The roots of the equation

$$\begin{bmatrix} x & \omega & \omega^2 \\ \omega^2 & x & \omega \\ \omega & \omega^2 & x \end{bmatrix} = 0 \text{ are}$$

(A) 1, –1, 2 (B) –1, –1, 2 (C) 1, –1, –2 (D) 1, 1, –2

21. $\lim_{n\to\infty} \dfrac{n^n + n!}{n^n}$

(A) does not exist (B) 1 (C) 0 (D) ∞

22. Suppose f: $[0, 1] \to R$ is given by

f(x) = 0 if x is rational and

$f(x) = \frac{1}{x}$ *if* x is irrational.

Then

(A) $\int_0^1 f(x)\, dx = 1$ (B) $\int_0^1 f(x)\, dx = 0$

(C) $\int_0^1 f(x)\, dx = \infty$. (D) f is not Riemann integrable

23. The centre of gravity of a circular cone of height h is

(A) $\frac{1}{4}h$ (B) h (C) $\frac{3}{4}h$ (D) $\frac{1}{2}h^2$

24. An open – top box is to be made by uniting small congruent squares from the corners of a $12'' X 12''$ sheet of tin and bending up the sides. How large should the squares cut from the corners be to make the box hold as much as possible?

(A) 4" (B) 2.5" (C) 3" (D) 2"

25. The linear fractional transformation that maps

$z_1 = 0, z_2 = 1, z_3 = \infty$ *onto* $w_1 = -1, w_2 = -i$ *and* $w_3 = 1$ respectively is

(A) z-i (B) $\dfrac{z-i}{z+i}$ (C) $\dfrac{z-1}{z+1}$ (D) e^z

26. The number of generators of a cyclic group of order 15 is

(A) 1 (B) 3 (C) 6 (D) 8

27. Let G be an additive group of integers modulo 24. The number of distinct subgroups of G is

(A) 24 (B) 12 (C) 8 (D) 1

28. Let (x_n) be a convergent sequence of real numbers with limit x. Then which one of the following statements is not true?

(A) (x_n) is monotonic

(B) (x_n) is bounded

(C) Every subsequence of (x_n) is convergent

(D) Every neighbourhood of x contains all but finitely many elements of (x_n)

29. It is found that the order of a group G is less than 320. If G has subgroups of orders 45 and 35, then the order of G is

(A) 80 (B) 105

(C) 315 (D) None of these

30. If three symmetrical dice are thrown, the probability that the sum of the numbers shown is 12 is

(A) $\frac{25}{216}$ (B) $\frac{12}{216}$ (C) $\frac{30}{216}$ (D) $\frac{36}{216}$

31. When x > a > 0, the integral $\int_a^\infty \frac{1}{x^p}\,dx$

(A) diverges if p > 1

(B) converges if p > 1

(C) diverges if p ≥ 1

(D) converges if p < 1 and diverges if p ≥ 1

32. Let f be a step function on [0,1] and

$$g(x) = \int_0^x f(t)\,dt \ \ for\ x \in [0.1].$$

If $E = \{x \in [0,1] : g \text{ is continuous at } x\}$ and

$F = \{x \in [0,1] : g \text{ is differentiable at } x\}$, then

(A) F = [0, 1]

(B) E = [0, 1] and F is an empty set

(C) E and F are empty sets

(D) E = [0, 1] but F is not an empty set

33. Let k € N. Let x be an irrational number with $0 < x < 1$ and $0.x_1x_2 ... x_i ...$ be the decimal expansion of x. Define a sequence (a_n) of reals as follows:

a_n = kth root of k if n = 2k

and $a_n = \sum_{i=1}^{k} \frac{x_i}{10^i}$ if $n = 2k - 1$.

Let $\alpha = \lim inf_n \ a_n$ and $\beta = \lim \sup_n a_n$. Then

(A) $0 < \beta - \alpha < 1 - x$

(B) $\beta - \alpha = 1 - x$

(C) $\alpha = \beta = 1$

(D) $\alpha = \beta = x$

34. Let f: R $\rightarrow$ R be a map given by

$f(x) = \{ \begin{smallmatrix} x\sin(1/x) & if \ x \neq 0 \\ 0 & if \ x = 0 \end{smallmatrix}$.

Then

(A) f is not continuous on R

(B) f is continuous and differentiable on R but f' is discontinuous

(C) f is continuous and differentiable on R and f' is continuous on R

(D) f is continuous on R but is not differentiable at 0

35. The solution of the differential equation $\frac{dy}{dx} = \frac{y^2}{1 - xy}$ is

(A) x y = ln y + c

(B) y = ln x + c

(C) y = c e^x

(D) y = c sin x

36. The differential equation corresponding to the general solution $y = Ae^{3x} + Be^{5x}$ is

(A) $y^{||} - 8y^{|} + 15y = 0$

(B) $y^{||} + 8y^{|} + 15y = 0$

(C) $y^{||} + 8y^{|} - 15y = 0$

(D) $y^{||} - 8y^{|} - 15y = 0$

37. The basis of solution of $y^{(4)} - 2y^{(3)} + 2y^{(2)} - 2y^{(1)} + y = 0$ is

(A) $e^x, xe^x, x^2e^x, x^3e^x$

(B) $e^x, xe^x, x^2e^x, \cos x$

(C) $e^x, xe^x, \cos x, \sin x$

(D) e^x, xe^x, x cos x, x sin x

38. The 2- step solution of the initial value problem $y^{|} = 2x - y,\ y(0) = 1$ is

(A) $1 + 3x^2 - x - x^3$

(B) $1 + \frac{3x^2}{2} - x - \frac{x^3}{3}$

(C) $1 + 2x + 3x^2 + 4x^3$

(D) $\sin x$

39. The particular solution of $\left(D^2 + 4\right) y = x \sin x$ is

(A) $\frac{1}{3}\left(x \sin x - \frac{2}{3} \cos x\right)$

(B) $\frac{x}{4} \sin x + \frac{x^2}{5} \cos x$

(C) $x \sin x - x^2 \cos x$

(D) $x \sin x + x^2 \cos x$

40. The general solution of $\frac{dx}{y+x} = \frac{dy}{z+x} = \frac{dz}{x+y}$ is

(A) $\varphi\left(x + y + z,\ x^2 - y^2\right)$

(B) $\varphi\left(\frac{y-x}{z-y},\ (x - y)^2 (x + y + z)\right)$

(C) $\varphi\left(\frac{x-y}{y-z},\ \frac{y-z}{z-x}\right)$

(D) $\varphi\left(\frac{x+y}{z},\ z\text{-}x - y\right)$

41. The Laplace transform of $e^{-t} \cos 2t$ is

(A) $\frac{s+1}{s^2+4s+5}$

(B) $\frac{s-1}{(s+1)^2}$

(C) $\frac{s^2-2s+3}{s^2+4s+5}$

(D) $\frac{s^2-1}{s^2+4s+5}$

42. The general solution of $y = xp + \sqrt{1+p^2}$, $p = y^{|}$ is

(A) $y = cx + \sqrt{1+c^2}$

(B) $y = cx + \sqrt{1+x^2}$

(C) $y = cx + \sqrt{1-x^2}$

(D) $y = cx + \sqrt{1-x}$

43. By which transformation, the Euler equation $x^2 y^{||} + xy^{|} + y = 0$ can be transformed into an equation with constant coefficients?.

(A) $Z = \log x$

(B) $Z = e^x$

(C) $Z = x^2$

(D) $x = z^2$

44. The angle of intersection at the point (2,-1,2) of the subspaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ is

(A) $\frac{8}{3}$ (B) $\frac{8}{2\sqrt{21}}$ (C) $\frac{7}{5}$ (D) 5

45. Suppose G is cyclic group which has exactly three subgroups viz, G, {e} and a subgroup of order 7. Then the order of G is

(A) 7 (B) 14 (C) 49 (D) 343

46. Let G be a group having two subgroups of order 14 and 24 respectively.

If o(G) < 300, then the order of G is

(A) 38 (B) 42 (C) 84 (D) 168

47. Let $\sum a_n$ be a series of positive terms. If $\sum a_n^2$ is convergent, then

(A) $\sum \frac{a_n}{n}$ diverges to $+\infty$

(B) $\sum \frac{a_n}{n}$ is convergent

(C) $\sum \frac{a_n}{n}$ oscillates finitely

(D) $\sum \frac{a_n}{n}$ oscillates infinitely

48. The function exp : $R \to R$ is

(A) injective

(B) monotonically increasing

(C) unbounded

(D) bounded

49. Let Z_{10} denote the ring of integers modulo 10. Then the number of ideals in Z_{10} is

(A) 2 (B) 3 (C) 4 (D) 5

50. If $f(x) = \begin{cases} x, \text{ for } x \text{ is rational} \\ 0, \text{ for } x \text{ is irrational} \end{cases}$ then f is

(A) discontinuous at all points of R

(B) continuous at all rational points

(C) continuous at all irrational points

(D) continuous only at zero

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(A) $\sum \frac{a_n}{n}$ diverges to $+\infty$ (B) $\sum \frac{a_n}{n}$ is convergent

(C) $\sum \frac{a_n}{n}$ oscillates finitely (D) $\sum \frac{a_n}{n}$ oscillates infinitely

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(A) injective (B) monotonically increasing

(C) unbounded (D) bounded

49. Let Z_{10} denote the ring of integers modulo 10. Then the number of ideals in Z_{10} is

(A) 2 (B) 3 (C) 4 (D) 5

50. If $f(x) = \begin{cases} x, \ for\ x\ is\ rational \\ 0, \ for\ x\ is\ irrational \end{cases}$ then f is

(A) discontinuous at all points of R (B) continuous at all rational points

(C) continuous at all irrational points (D) continuous only at zero

51. If $f : R \to R$ is a map then

(A) f is differentiable on R if $|f|$ is differentiable on R

(B) $|f|$ is differentiable on R if f is differentiable on R

(C) f is continuous on R if $|f|$ is continuous on R

(D) None of these

52. If $a_n = Cos\frac{np}{2}$ for n^3 1, then

(A) lim sup n ® ¥ an = 0 and lim inf an = 0

(B) lim sup n ® ¥ an = 1 and lim inf an = 0

(C) lim sup n ® ¥ an = 0 and lim inf an = −1

(D) lim sup n ® ¥ an = 1 and lim inf an = −1

53. If the determinant $\begin{vmatrix} x- h & 0 & h- x \\ 2x & x+ h & 2x \\ x+ h & 2h & x+ h \end{vmatrix}$ is of the form $k(x- \ y)^3$, then the pair (k,y) is

(A) (1,h) (B) (1,2h) (C) (2,h) (D) (h,2)

54. The solution of $\frac{dy}{dx} = e^{2x+y}$ is

(A) $\frac{e^{2x}}{2} + e^{y} = k$

(B) $\frac{e^{2x}}{2} - e^{-y} = k$

(C) $\frac{e^{2x}}{2} + e^{-y} = k$

(D) $y = \frac{e^{2x+y}}{2} + k$

55. Let A and B be two non-empty sets. A relation from A to B is a subset of

(A) $A \cup B$ (B) $A \cap B$ (C) $A - B$ (D) $A \times B$

56. Let A and B be sets such that $|A| = m$ and $|B| = n$. The set of all functions from A to B is denoted by B^A. Then $|B^A|$ is

(A) $m + n$ (B) mn (C) m^n (D) n^m

57. The function $f : R \to R$ defined by the rule $f(x) = 3x$ is

(A) bijection
(B) 1 - 1 but not onto
(C) not 1 - 1 but onto
(D) neither 1- 1 nor onto

58. Let $f : A \to B$ and $g : B \to C$ be two functions such that $f \circ g : A \to C$ is a bijection. Then

(A) f is onto and g is 1-1
(B) f is 1-1 and g is onto
(C) f and g are bijections
(D) none of these

59. Let $G = \left\{ \begin{bmatrix} x & x \\ x & x \end{bmatrix} / x \in R^* \right\}$. Under the matrix multiplication G is

(A) a group with $e = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
(B) not a group
(C) an abelian group
(D) a non abelian group

60. The order of the element 3 in the additive group Z_8 is

(A) 2
(B) 4
(C) 6
(D) 8

61. Which one of the following is a group?

(A) The set of all even integers under addition
(B) The set of all even integers under subtraction
(C) The set of all odd integers under addition
(D) The set of all odd under subtraction

62. Let $p = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 1 & 2 \end{pmatrix}$ be a permutation. Then the inverse of p is

(A) $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix}$
(B) $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 1 & 2 \end{pmatrix}$
(C) $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 2 & 1 & 3 \end{pmatrix}$
(D) $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 1 & 4 \end{pmatrix}$

63. The order of -1 in $(Z, +)$ is

(A) 2
(B) 1
(C) 0
(D) infinite

64. The kernel of the homomorphism $f : (Z, +) \to (R^*, .)$ defined by $f(x) = 2^x$ is

(A) $\{1\}$
(B) $\{0\}$
(C) Z
(D) $\{1, -1\}$

65. The algebraic structure which is not a ring is

(A) (Z,+, .) (B) (Q,+, .) (C) (R,+, .) (D) (R, ., +)

66. The character of (Z7,L,N) is

(A) 7 (B) 1 (C) 0 (D) 2

67. The function f : Z →Z defined by the rule f(x) = 2x is a ...

(A) ring homomorphism (B) ring isomorphism

(C) ring automorphism (D) group homomorphism

68. In R^3 let S = $\{e_1, e_2, e_3\}$. Then L(S) is

(A) S (B) {(x, y, 0) / x, y € R}

(C) {(0, y, z) / y, z € R} (D) R^3

69. In R^3 let S = L({(1, 1, 1)}) and T = L({(−1,−1,−1)}) . Then dim S∩T is

(A) 1 (B) 0 (C) 2 (D) 4

70. If K is a compact subset of R then W = {|x| : x belongs to K} is

(A) R (B) a compact set in R

(C) [0, ∞) (D) a closed set but not compact in R

71. The set of 2 × 2 matrices with trace 0 is

(A) a group under addition but not a vector subspace

(B) a non-commutative group under multiplication

(C) a commutative group under multiplication

(D) a vector subspace

72. A and B are m × n matrices. Then the size of $A^T + B^T$ is

(A) m × n (B) m × m (C) n × n (D) n × m

73. If the entries of a 2 × 2 matrix A are defined by the formula $a_{ij} = i^2 + j^2$, then A is

(A) a symmetric matrix (B) a skew-symmetric matrix

(C) the identity matrix (D) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

74. If A = $\begin{bmatrix} 1 & 2 \\ 3 & n \end{bmatrix}$ and B = $\begin{bmatrix} m & 2 \\ n & 4 \end{bmatrix}$ are singular matrices then the value of mn is

(A) 2 (B) 3 (C) 18 (D) 6

75. The inverse of $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ is

(A) $\begin{bmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix}$

(B) $\begin{bmatrix} 2 & 1 \\ -\frac{3}{2} & \frac{1}{2} \end{bmatrix}$

(C) $\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$

(D) $\begin{bmatrix} \frac{1}{2} & 1 \\ 2 & -\frac{3}{2} \end{bmatrix}$

76. The rank of the matrix $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ is

(A) 1 (B) 2 (C) 3 (D) 4

77. The characteristic roots of $\begin{bmatrix} \cos Ø & \sin Ø \\ -\sin Ø & \cos Ø \end{bmatrix}$

(A) –1, –1

(B) 1, 1

(C) cos Ø – sin Ø; cos Ø + sin Ø

(D) sin Ø – cos Ø ; sin Ø + cos Ø

78. The product of the eigen values of $\begin{bmatrix} 8 & -6 & -6 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ is

(A) 120 (B) 40 (C) 18 (D) –40

79. The quadratic form in two variables x_1, x_2 for the symmetric matrix $\begin{bmatrix} 1 & -2 \\ -2 & -1 \end{bmatrix}$ is

(A) $x_1^2 - x_2^2 - 2x_1x_2$

(B) $x_1^2 - x_2^2 - 4x_1x_2$

(C) $x_1^2 - x_2^2 + 2x_1x_2$

(D) $(x_1 + x_2)^2$

80. If $y^x = x^{\sin y}$, then $\frac{dy}{dx}$ is

(A) $\frac{\sin y}{x \cos y}$

(B) $\frac{\log y - \frac{\sin y}{x}}{\cos y \log x - \frac{x}{y}}$

(C) x cot y

(D) $\frac{\sin y - \log y}{\cos y - \log x}$

81. The differential coefficient of $tan^{-1} \frac{x}{\sqrt{1-x^2}}$ with respect to $sec^{-1} \frac{1}{2x^2-1}$ is

(A) x (B) 1 (C) $\frac{x}{2}$ (D) $-\frac{1}{2}$

82. If $p^2 = a^2 cos^2\vartheta + b^2 sin^2 \vartheta$, then $p + \frac{d^2p}{d\vartheta^2}$ is

(A) $a^2b^2p^2$ (B) $\frac{a^2b^2}{p^3}$ (C) $a^2b^2p^4$ (D) $\frac{1}{a^2b^2p^2}$

83. $\frac{x}{2} - \frac{x^3}{6} + \frac{x^4}{10} - \frac{x^7}{14} + ...$ is

(A) $e^{x/2}$

(B) $\sin\frac{x}{2}$

(C) $\tan\frac{x}{2}$

(D) $tan^{-1} \frac{(\sqrt{1+x^2})-1}{x}$

84. If $y = \sqrt{sinx + \sqrt{sinx + \sqrt{sinx + }}}$.....to ∞,

then $\frac{dy}{dx}$ is

(A) $y\sqrt{cosx}$ (B) y cosx (C) $\frac{1}{y}$ cosx (D) $\frac{cosx}{2y-1}$

85. The solution of the equation

$tan^{-1}(x+1) + tan^{-1}(x-1) = tan^{-1}\frac{8}{31}$ is

(A) $\frac{1}{4}$ (B) –8 (C) $\sqrt{2}$ (D) $2\sqrt{2}$

86. $\lim_{\vartheta \to 0} \frac{\tan\vartheta + \sec\vartheta - 1}{\tan\vartheta - \sec\vartheta + 1}$ is

(A) 0 (B) 1 (C) $\frac{\pi}{2}$ (D) $n\pi$

87. If sin A + sin B + sin C = Cos A + Cos B + Cos C = 0

then cos^2 A + cos^2 B + cos^2 C is

(A) π (B) $\frac{\pi}{2}$ (C) $\frac{1}{2}$ (D) $\frac{3}{2}$

88. The points B, P, Q in the Argand diagram represent the complex numbers 2, z, z^2. If P describes the circle on AB as diameter, the locus of Q is

(A) Circle (B) Ellipse (C) Cardioid (D) Hypocycloid

89. If $\frac{\sin\theta}{\theta} = \frac{5045}{5046}$ then the approximate value of θ is

(A) $0^\circ\ 32'$ (B) 1° (C) $1^\circ\ 58'$ (D) $2^\circ\ 32'$

90. If $\tan\frac{x}{2} = \tan h\frac{y}{2}$ then $\log\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)$ is

(A) $\frac{y}{x}$ (B) y (C) x^2-y^2 (D) x^2+y^2

91. The real matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ will have real eigen values when

(A) $(a-d)^2 + 4bc \geq 0$ (B) $(a-d)^2 + 4bc \leq 0$

(C) $(a-d)^2 - 4bc \geq 0$ (D) $(a-d)^2 - 4bc \leq 0$

92. Let T: $R^2 \to R^3$ be a linear transformation for which T(1, 2) = (3, –1, 5) and T(0, 1) = (2, 1, -1). Then T(x, y) is

(A) (–x+2y, –3x+y, 7x+y) (B) (–x+2y, 3x+y, 7x-y)

(C) (x+2y, –3x+y, 7x-y) (D) (–x+2y, –3x+y, 7x–y)

93. If Sin (A + i B) = tan C + i Sec D , then

Cos 2A Cos h 2B is

(A) 1 (B) 2 (C) 3 (D) 4

94. $\int_0^{\frac{\pi}{2}} \frac{\cos\theta}{1+\sin^2\theta}\, d\vartheta$ is

(A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) 2π (D) $\frac{\pi}{4}$

95. Let M be a metric space. Let f : M → R and g : M → R be continuous functions. Which one of the following statements is not true?

(A) f + g is continuous (B) fg is continuous

(C) cf is continuous for all c in R (D) f/g is continuous

96. If $f(x) = x\,e^{x}$ then

(A) f is unbounded from below

(B) $f(x) > -\frac{1}{e}$ for every x

(C) f is a bounded function

(D) $f(x) < -\frac{1}{e}$ for every x

97. The function cos h x on R is

(A) continuous but not differentiable

(B) differentiable but not bounded

(C) bounded but not continuous

(D) bounded

98. Two sets A and B are equivalent if there exists a mapping from A to B which is

(A) an injection

(B) a bijection

(C) a surjection

(D) a homomorphism

99. Which is the incorrect statement?

(A) (0, 1] is uncountable

(B) [0, 1] is uncountable

(C) $Q \times (0, 1)$ is uncountable

(D) $\{0\}\cup\{1\}\cup\{2\}$ is uncountable

100. Which is the incorrect statement in any metric space (M, d)?

(A) Ø is open

(B) the union of any family of open sets is open

(C) M is open

(D) the intersection of any number of open sets is open