ENTRANCE EXAMINATION FOR ADMISSION, MAY 2011.

M.Sc. (MATHEMATICS)

COURSE CODE : 372

Register Number :

Signature of the Invigilator
(with date)

COURSE CODE : 372

Time : 2 Hours — Max : 400 Marks

Instructions to Candidates :

1. Write your Register Number within the box provided on the top of this page and fill in the page 1 of the answer sheet using pen.
2. Do not write your name anywhere in this booklet or answer sheet. Violation of this entails disqualification.
3. Read each question carefully and shade the relevant answer (A) or (B) or (C) or (D) in the relevant box of the ANSWER SHEET using HB pencil.
4. Avoid blind guessing. A wrong answer will fetch you –1 mark and the correct answer will fetch 4 marks.
5. Do not write anything in the question paper. Use the white sheets attached at the end for rough works.
6. Do not open the question paper until the start signal is given.
7. Do not attempt to answer after stop signal is given. Any such attempt will disqualify your candidature.
8. On stop signal, keep the question paper and the answer sheet on your table and wait for the invigilator to collect them.
9. Use of Calculators, Tables, etc. are prohibited.

Notation : $\mathbb{R}$ – Real line, Q – Set of rationals $\mathbb{N}$ - Natural Numbers, $\mathbb{C}$ - Complex plane $\overline{A}$ – Closure of A, A^c -Complement of A *and* ϕ -empty set, $i = +\sqrt{-1}, A^T$ – Transpose of Matrix A .

1. For any finite set E, let $|E|$ denote the number of elements in E. If A and B are finite sets and then $|A \cup B| + |A \cap B| =$

   (A) $|A| + |B|$ (B) $||A| - |B||$ (C) $\geq |A| + |B|$ (D) $< |A| + |B|$

2. Let X *and* Y be subsets of a topological space and let $\overline{A}$ denote the closure of a subset A of the topological space. Then, $\overline{X \cup Y} =$

   (A) $\overline{X} \setminus \overline{Y}$ (B) $\overline{X} \cup \overline{Y}$ (C) $\overline{X} \cap \overline{Y}$ (D) $\overline{Y} \setminus \overline{X}$

3. A set, $A \subset \mathbb{R}$ is compact if and only if A is

   (A) open and bounded (B) closed and bounded
   (C) open and unbounded (D) closed and unbounded

4. A set $\{x \in \mathbb{R} : 4 \leq x^2 \leq 9\}$ is

   (A) open (B) compact and connected
   (C) compact and disconnected (D) disconnected but not compact

5. Let $f(x)$ be the function defined for all

   $x \in \mathbb{R}$ *such that* $f(0) = 0$ *and* $f(x) = x\sin(\frac{1}{x})$ *for* $x \neq 0$. *Then* f *is*

   (A) continuous only at $x = 0$ (B) discontinuous at $x = 0$
   (C) discontinuous at $x = 1$ (D) continuous on $\mathbf{R}$

6. The value of C for which the limit $\lim_{x \to +\infty} \{(x^5 + 7x^4 + 2)^c - x\}$ is finite and non zero is

   (A) -1 (B) $\frac{1}{5}$ (C) 0 (D) 1

7. If $z = \frac{1+i}{i-2}$ then the real part of z is

   (A) $\frac{1}{2}$ (B) $\frac{-1}{5}$ (C) $\frac{-1}{2}$ (D) $\frac{1}{5}$

8. If $f(z) = |z|^2$ *then*

   (A) f satisfies the Cauchy Riemann equation at all $z \in \mathbf{C}$
   (B) f satisfies the Cauchy Riemann equation only at $z = -i$
   (C) f satisfies the Cauchy Riemann equation only at $z = 0$
   (D) f satisfies the Cauchy Riemann equation only at $z = i$

9. Let f *and* g be two real valued functions, which have continuous derivatives on (a,b). Then $\int_a^b f(x)g'(x)dx + \int_a^b g(x)f'(x)dx$ *is*

(A) $f(a)g(a)-f(b)g(b)$
(B) $f(b)g(b)-f(a)g(a)$
(C) $f(a)g(b)-f(b)g(a)$
(D) $f(b)g(a)-f(a)g(b)$

10. If x is real, then $\lim_{x\to 0}\frac{3\sin x-\sin 3x}{x-\sin x}$ is

(A) 0
(B) π
(C) 1
(D) 24

11. The graph of the curve $x^2-y^2=1$ *is known as*

(A) ellipse
(B) cycloid
(C) parabola
(D) hyperbola

12. Let f *and* g be continuous functions on $[a,b]$ *such that* $f(a)<g(a)$ *and* $f(b)>g(b)$. Then the set $\{x\in[a,b]: f(x)=g(x)\}$

(A) equals $[a,b]$
(B) is a non-empty proper subset of $[a,b]$
(C) empty
(D) must be an infinite set

13. $\int \frac{dx}{a^2\cos^2 x+b^2\sin^2 x}$ *is*

(A) $\tan(\frac{bx}{a})$
(B) $\frac{1}{ab}\tan^{-1}(\frac{bx}{a})$
(C) $\frac{1}{ab}\tan^{-1}(\frac{b\tan x}{a})$
(D) $\sec(\frac{bx}{a})$

14. If f *and* g are differentiable functions from $\mathbb{R}$ *into* $\mathbb{R}$ *such that* $f(0)=\frac{1}{g(0)}$ *and* $h(x)=f(x)\ g(x)\sin\frac{\pi}{2}x$ *then* $h^1(0)$ *is*

(A) $\frac{2}{\pi}$
(B) $\frac{4}{\pi}$
(C) $\frac{-2}{\pi}$
(D) 1

15. *T* the Argand diagram of the points $5+2i$, $7+5i$, $4+7i$ *and* $2+4i$ *form a*

(A) parallelogram but not a rectangle
(B) square
(C) rectangle but not a square
(D) rhombus but not a square

16. The directional derivative of the function $f(x,y,z)=xy^2z$ at the point (1,0,0) in the direction $(2\vec{i}-2\vec{j}+\vec{k})$ is

(A) 1
(B) –1
(C) 0
(D) –2

17. For the function $f(z)=e^{\frac{1}{z}}$, *the point* $z=0$,

(A) a removable singularity
(B) not a singularity
(C) an essential singularity
(D) a simply pole

18. If C is the circle $|z|=2$, *the value of* $\int_c \frac{e^{-z}}{z^2}dz$ *is*

(A) 0 (B) -1 (C) $-2\pi i$ (D) $2\pi i$

19. If (x_n) is a sequence of non negative real numbers then $\limsup_n(-x_n)$ *is*

(A) $-\limsup_n(x_n)$ (B) $-\liminf_n(x_n)$

(C) $\liminf_n(x_n)$ (D) $\limsup(x_n)$

20. Let $f:G \to H$ is a group homomorphism. Then G is isomorphic to $f(G)$

(A) if and only if f is onto (B) if and only if f is $1-1$

(C) only if f is an isomorphism (D) if G is cyclic

21. Let $f(z):\mathbb{C}\to\mathbb{C}$ be analytic function such that $0\leq|f(z)|<1$. Then

(A) $f(z)$ *is a constant for all* $z\in\mathbb{C}$ (B) $f(z)=0$ *for all* $z\in\mathbb{C}$

(C) $|f(z)|\to 0$ *as* $z\to\infty$ (D) $|f(z)|\to 1$ *as* $z\to\infty$

22. Let R be a ring and *a, b* be invertible elements of R. Then the product,

(A) *ab* is invertible (B) *ab* is invertible if R is commutative ring

(C) *ab* is invertible but *ba* need not be invertible (D) *ab* is invertible if R is a field

23. If $tan^{-1}x + tan^{-1}y + tan^{-1}z = \frac{\pi}{2}$, then $xy + yz + zx$ is

(A) 0 (B) -1 (C) π (D) 1

24. If $f:\mathbb{R}\to\mathbb{R}$ *defined by* $f(x)=1+ax$, *where* $a\neq 0$, is such that $f=f^{-1}$. Then the value of a is

(A) -2 (B) -1 (C) 1 (D) 2

25. If $\lim_{x\to 0}\frac{f(x)}{x^2}=1$, *then* $\lim_{x\to 0}\frac{f(x)}{x}$ *is*

(A) 1 (B) 0 (C) -1 (D) ∞

26. Let ρ be a relation on a collection of sets defined by $A\rho B \Leftrightarrow A\cap B=\phi$. *Then* ρ *is*

(A) an equivalence relation (B) reflexive and symmetric only

(C) symmetric and transitive only (D) not reflexive

27. The relation ρ is defined on $\mathbb{Z}$ *as* $x\rho y \Leftrightarrow x-y$ is a multiple of 5. Then the equivalence class to which 2 belongs is the set

(A) $\{5k+2/k\in\mathbb{Z}\}$ (B) $\{2k+5/k\in\mathbb{Z}\}$

(C) $\{2k+5/k\in\mathbb{N}\}$ (D) $\{5k+2/k\in\mathbb{N}\}$

28. Let A be a finite set of size of n. The number of elements in the power set of $A \times A$ is

(A) 2^{2n} (B) $(2n)^2$ (C) 2^{n^2} (D) $2n^2$

29. Let A and B be sets such that $|A| = m$ and $|B| = n \geq m$, The number of 1-1 functions from A and B is

(A) $n(n-1)(n-2)...(n-m+1)$ (B) mn

(C) m^n (D) n^m

30. The inverse of the matrix $A = \begin{bmatrix} 5 & 4 \\ 6 & 5 \end{bmatrix}$

(A) does not exist (B) $\begin{bmatrix} 5 & -4 \\ 6 & 5 \end{bmatrix}$

(C) $\begin{bmatrix} 5 & 4 \\ -6 & 5 \end{bmatrix}$ (D) $\begin{bmatrix} 5 & -4 \\ -6 & 5 \end{bmatrix}$

31. The system of equations

$$x + y + z = 2$$
$$2x + 3z = 5$$
$$3x + y + 4z = 6$$

(A) has no solution for (x, y, z)

(B) has a unique solution for (x, y, z)

(C) has more than one but finite number of solutions for (x, y, z)

(D) has infinite number of solutions for (x, y, z)

32. If $T : \mathbb{R}^4 \to \mathbb{R}^4$ *is given by* $T(x_1, x_2, x_3, x_4) = (x_1^2, x_2, 0, x_4)$ *for* $(x_1, x_2, x_3, x_4) \in \mathbb{R}^4$ *then,*

(A) T is linear and 1 – 1 (B) T is linear and onto

(C) T is linear but not onto (D) T is not linear

33. Mark the **wrong** statement

If A *is the matrix* $\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$

(A) there is a matrix P such that PAP^{-1} is diagonal

(B) the eigenvectors of A *span* $\mathbb{R}^2$

(C) the eigenvectors of A are linearly dependent

(D) there is a 2 x 2 matrix B such that AB=Ba=I where I is the identity matrix

34. Choose the matrix for which the inverse exists

(A) $\begin{pmatrix} 2 & 1.5 \\ 4 & 3 \end{pmatrix}$ (B) $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ (C) $\begin{pmatrix} 3 & 3 \\ 2 & 2 \end{pmatrix}$ (D) $\begin{pmatrix} \frac{1}{10} & \frac{2}{5} \\ \frac{1}{20} & \frac{1}{5} \end{pmatrix}$

35. If the eigen values of $A=\begin{pmatrix}1 & 1 & 3\\ 1 & 5 & 1\\ 3 & 1 & 1\end{pmatrix}$ are -2, 3, 6, then the eigen values of the transpose A^T are

(A) $-2,3,6$ (B) $\frac{1}{2},\frac{1}{3},\frac{1}{6}$ (C) $-2^2,3^2,6^2$ (D) $-4,6,12$

36. If $A=\begin{bmatrix}2 & 0 & 0\\ 0 & -1 & 0\\ 0 & 0 & 5\end{bmatrix}$ then

(A) A is not Hermitian
(B) A is orthogonal
(C) A is singular
(D) $Ax = b$, where b is a non-zero vector in $\mathbb{R}^3$, has only the zero solution for $X \in \mathbb{R}^3$

37. The sum of the characteristic roots of $\begin{pmatrix}\cos\theta & -\sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}$ are

(A) 0 (B) 1 (C) $2\cos\theta$ (D) $2\sin\theta$

38. The area under the curve $y=\frac{1}{x}$ between the ordinates at $x=1$ *and* $x=2$ *is*

(A) $\frac{1}{2}$ (B) $\log 2$ (C) $e-1$ (D) 1

39. The value of $\sin\frac{\pi}{12}$ is

(A) $\frac{\sqrt{3}-1}{2}$ (B) $\frac{\sqrt{3}+1}{2}$ (C) $\frac{\sqrt{3}+1}{\sqrt{2}}$ (D) $\frac{\sqrt{3}-1}{2\sqrt{2}}$

40. The set of points on $\mathbb{R}$ where the function $y=\frac{x\tan x}{x^2+1}$ is continuous equals

(A) $\mathbb{R}$
(B) $\mathbb{R}\setminus\{\frac{n\pi}{2}: n \textit{ is an even integer}\}$
(C) $\mathbb{R}\setminus\{\frac{n\pi}{2}: n \textit{ is an odd integer}\}$
(D) $\mathbb{R}\setminus\{\frac{n\pi}{2}: n \textit{ is any integer}\}$

41. $\int_1^e \frac{\log x}{x}dx$ *equals*

(A) 0 (B) 1 (C) $\frac{1}{2}$ (D) e

42. If $i^{x+iy} = a+ib$, *then* a^2+b^2 *is*

(A) $e^{-\pi y}$ (B) $e^{-\pi x}$ (C) $e^{\pi x}$ (D) $e^{\pi y}$

43. A particular solution of the equation $y''+y=\sin x$ is given by

(A) $\sin x$ (B) $\cos x$ (C) $\frac{-x\cos x}{2}$ (D) $\frac{-x\sin x}{2}$

44. Mark the **wrong** statement :

A solution of the equation $x^2y''+xy'-y=0$ is given by

(A) $y(x)=1$ (B) $y(x)=x$ (C) $y(x)=x^{-1}$ (D) $y(x)=0$

45. A solution of the equation $x^2y''-3xy'-4y=0$ *is given by* $y(x)=$

(A) x (B) x^2 (C) $x\log x$ (D) xe^x

46. A solution of the equation $y''-4y'+4y=0$ *is given by* $y(x)=$

(A) x (B) xe^x (C) x^2e^{2x} (D) e^{2x}

47. A solution of the equation $y''-2y'+10y=0$ *is given by* $y(x)=$

(A) $e^{x\cos 3x}$ (B) $\cos 3x$

(C) $\sin 3x$ (D) $\sin 3x+\cos 3x$

48. The rank of the matrix $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ is

(A) 1 (B) 2 (C) 3 (D) 4

49. $\lim_{x\to 0} \frac{1}{x}\log(1+x)$ is

(A) 0 (B) 1 (C) –1 (D) ∞

50. $\lim_{x\to 0} \frac{\sin^2 x}{\sin x^2}$ is

(A) 0 (B) π (C) ∞ (D) 1

51. If $f(x)=\begin{cases}1-2x \text{ if } x\leq 1\\ 3x-4 \text{ if } x>1\end{cases}$ then

(A) f is continuous at all points except at $x=1$

(B) f is continuous on **R** but not differentiable at any point of **R**

(C) f is continuous on **R** and differentiable at all points except $x=1$

(D) f is discontinuous at $x=1$

52. If $f(x)=e^x$ *and* $g(x)=x+1$ *for all* $x\in\mathbb{R}$ then

(A) $(f-g)(x)\geq 0$ *for all* $x\in\mathbb{R}$

(B) $(f-g)(x)\geq 0$ *for all* $x\in[0,\infty]$ *and* $(f-g)(x)\leq 0$ *for* $x\in(-\infty,0]$

(C) $(f-g)(x)\leq 0$ *for* $x\in(-\infty,-1]$

(D) $(f-g)(x)\leq 0$ *for all* $x\in\mathbb{R}$

53. $f(x)=\begin{cases}x^2+1 & \text{if } 0\leq x\leq 1\\ 2e^{x-1} & \text{if } \quad x>1\end{cases}$

(A) is continuous and monotone increasing on $\mathbb{R}$

(B) is discontinuous at $x=1$

(C) is continuous but not monotone increasing on $\mathbb{R}$

(D) is monotone increasing but not continuous on $\mathbb{R}$

54. $\int_{-\frac{3\pi}{2}}^{\frac{3\pi}{2}} 6x^{16}\sin 5x\,dx$ equals

(A) $\sin\frac{\pi}{2}$ (B) $\frac{3\pi}{2}\sin\frac{5\pi}{2}$ (C) 0 (D) $-\pi$

55. $\int_1^2 \frac{(\log x)^2}{x}dx$ equals

(A) 1 (B) $(\log 2)^2$ (C) 0 (D) $\frac{(\log 2)^3}{3}$

56. The tangent to the curve $y=3x^2-1$ *at* $(0,-1)$

(A) is parallel to the X-axis (B) is parallel to the Y-axis

(C) has slope 1/3 (D) has slope 3

57. If the fourth derivative of a map $f:\mathbb{R}\to\mathbb{R}$ *exists on* $\mathbb{R}$ *and* f *has* 5 distinct zeros, then the third derivative of f has

(A) atleast 3 distinct zeroes (B) atleast 4 distinct zeroes

(C) atleast 2 distinct zeroes (D) need not have any zeroes

58. The map $f(\theta)=\cos(i\theta),\theta\in\mathbb{R}$

(A) is a bounded function (B) $f(1)$ is not a real number

(C) f *is a* $2\pi-$ periodic function (D) is an even function

59. If $A=\{(x,y):|x|+|y|\leq 1\}$ then

(A) $(\frac{-1}{\sqrt{3}},\frac{-2}{\sqrt{3}})\in A$ (B) $(-1,1)\in A$ (C) $(\frac{-1}{\sqrt{2}},\frac{1}{\sqrt{2}})\in A$ (D) $(\frac{-1}{3},\frac{-2}{3})\in A$

60. If $A=\{(x,y):y=0\}$ *and* $B=\{(x,y):xy=1\}$ *then*

(A) $A\cap B=\phi$ (B) $A\cup B=\mathbb{R}^2$

(C) $A\cap B$ is a non empty finite set (D) $A\cap B$ is a non empty infinite set

61. If A and B are square $n\times n$ matrices and A is singular then

(A) Rank of $AB=n$ (B) AB is singular

(C) AB is non-singular if B is non-singular (D) None of these

62. The value of the determinant $\begin{vmatrix} 7 & -21 & 89 & 1 \\ -31 & 93 & 33 & -1 \\ 56 & -168 & 1 & -1 \\ -74 & 222 & 0 & 5 \end{vmatrix}$ is

(A) 5 (B) 1 (C) -1 (D) 0

63. The series $\sum_{n=1}^{\infty}\frac{kn}{n^3+100}$, *where k is real*,

(A) is convergent for any real number k (B) is convergent only if $k\leq 100$

(C) is divergent if $k>1$ (D) is divergent if $k<0$

64. The radius of convergence of the power series $\sum_{n=1}^{\infty}\frac{z^n}{n!}$ *where* $z\in\mathbb{C}$, *is*

(A) ∞ (B) 1 (C) 0 (D) 2

65. If $f(x)=\begin{cases}1 & if \quad x<-1\\ -x & if\ -1\le x<0\\ x & if\ \ 0\le x\le 1\\ 1 & if \qquad x>1\end{cases}$ then

(A) f is continuous on $\mathbb{R}$

(B) f is discontinuous at 1

(C) f is discontinuous at –1

(D) f is continuous at 0

66. The curve $y=\log_e 3x, x>0$,

(A) intersects the X-axis at $x=3$ *and* $x=\frac{1}{3}$

(B) intersects the X-axis at $x=\frac{1}{3}$

(C) intersects the X-axis $x=1$

(D) does not intersect the X-axis

67. Let $A=(x\in[0,1]: x\ has\ more\ than\ one\ decimal\ expansion\}$. Then

(A) $A=\phi$

(B) $A=[0,1]$

(C) A is non-empty and countable

(D) A is an uncountable set

68. Let A be $n\times n$ non-singular matrix, with entries from $\mathbf{R}$. If $T:\mathbf{R}^n\to\mathbf{R}^n$ is the linear operator represented by A then

(A) T is 1 – 1 but not onto

(B) T is onto but not 1–1

(C) T is 1–1 and onto

(D) T is neither 1–1 nor onto

69. If V is a vector space and $T:V\to V$ is a linear map then

(A) $T^2:V\to V$ is a linear map

(B) $T^3:V\to V$ is not linear map

(C) $T^4:V\to V$ is not a linear map

(D) If T^{-1} exists, it need not be linear

70. The set $A=\{(r_1,r_2,r_3): r_i\ \ is\ a\ rational\ \ for\ 1\le i\le 3\}$

(A) is finite

(B) is countably infinite and dense in $\mathbf{R}^3$

(C) is infinite and uncountable

(D) is countably infinite but not dense in $\mathbf{R}^3$

71. If $x=(-1,2,1),\ y=(2,2,-3),\ z=(1,2,-2)$ *are elements of* $\mathbb{R}^3$, then

(A) *x is orthogonal to $y+z$*

(B) *y is orthogonal to $z+x$*

(C) *z is orthogonal to $x+y$*

(D) *x is orthogonal to $x+y+z$*

72. If $f(x)=\begin{cases}\sin\frac{1}{x} & if\ x\neq 0\\ 0 & if\ x=0\end{cases}$ and $A^{+}=\{x\in[0,1]:f(x)=1\}$ *and* $A^{-}=\{x\in[0,1]:f(x)=-1\}$.

Then

(A) Both A^{+} *and* A^{-} are empty sets

(B) A^{+} *and* A^{-} are finite sets

(C) $A^{+}\cup A^{-}=[0,1]$

(D) A^{+} *and* A^{-} are both infinite sets

73. If $a=\sup_{n} a_n$ *where* $(a_n)_{n=1}^{\infty}$ *is a sequence of reals and* $a>a_n$ for each n *and* $b<a<c$. *Then*

(A) $\{n\in\mathbb{N}:b<a_n<a\}$ is a non-empty finite set

(B) $\{n\in\mathbb{N}:b<a_n<a\}$ is an infinite set

(C) $\{n\in\mathbb{N}:a_n\geq a\}$ is non-empty

(D) $\{n\in\mathbb{N}:b<a_n<a\}$ is empty

74. If $f:\mathbb{R}\to\mathbb{R}$ *is continuous*, $f^{+}=\max\{f,0\}$ *and* $f^{-}=\max\{-f,0\}$, *then*

(A) f *is continuous if* f^{+} *is continuous*

(B) f *is continuous if* f^{-} *is continuous*

(C) f^{+} *and* f^{-} *are continuous if* f *is continuous*

(D) f *is continuous does not imply* f^{+} *or* f^{-} *is continuous*

75. The function exp : $R\to R$ is

(A) injective

(B) monotonically increasing

(C) unbounded

(D) bounded

76. The inverse of $\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}$ is

(A) $\begin{bmatrix}-2 & 1\\ \frac{3}{2} & -\frac{1}{2}\end{bmatrix}$

(B) $\begin{bmatrix}2 & 1\\ -\frac{3}{2} & \frac{1}{2}\end{bmatrix}$

(C) $\begin{bmatrix}4 & -2\\ -3 & 1\end{bmatrix}$

(D) $\begin{bmatrix}\frac{1}{2} & 1\\ 2 & -\frac{3}{2}\end{bmatrix}$

77. If $T:\mathbb{R}^3\to\mathbb{R}^3$ *is given by* $T(x_1,x_2,x_3)=(x_1+x_2,0,x_3)$ *for* $(x_1,x_2,x_3)\in\mathbb{R}^3$ *then*

(A) T is linear and onto

(B) T is linear but not onto

(C) T is linear and $1-1$

(D) T is not linear

78. If T is a linear map from
$\mathbb{R}^3$ *into* $\mathbb{R}$ *and* $T(1,0,0)=2$, $T(0,1,0)=-1$ *and* $T(0,0,1)=3$. *Then* $T(-5,2,3)$ *is*
(A) -3 (B) 3 (C) 2 (D) 1

79. Let $T_i = V \to V$ *be a linear map where* V *is a vector space for* $i=1,2$. If x *is an eigenvalue of* T_1 *and* T_2. *Then*
(A) $2x$ *is an eigenvalue of* T_1+T_2
(B) x *is an eigenvalue of* T_1+T_2
(C) $4x$ *is an eigenvalue of* T_1+T_2
(D) 0 is an eigenvalue of T_1+T_2

80. If x *and* y are elements of 3-dimensional vector space V *and* $y \notin span(x)$ *then*
(A) $\{x, y, x+y)$ *form a basis of* V
(B) $\{x, y, x-y\}$ *form a basis of* V
(C) $\{y-x, 2x-y, x\}$ *form a basis of* V
(D) $\{x+y, x-y, y\}$ *does not form a basis of* V

81. If $\vec{a} = a_1 i + a_2 j + a_3 k, \vec{b} = b_1 i + b_2 j + b_3 k$ *are two vectors in* $\mathbb{R}^3$, then the vector $\vec{a} \times \vec{b}$ is perpendicular to $k\vec{a} + m\vec{b}$ for
(A) *any* $k \in \mathbb{R}$ *and* $m \in \mathbb{R}$
(B) *only when* $k \geq 0$ *and* $m \geq 0$
(C) *only when* $k=0$ *and* $m=1$ *or* $k=1$ *and* $m=0$
(D) *only when* $k=m=0$

82. If $\vec{a}, \vec{b}$ *and* $\vec{c}$ *are* 3 *vectors in* $\mathbb{R}^3$ *and* $(\vec{a} \times \vec{b}).\vec{c} = 0$. *Then*
(A) $\vec{c}$ *is parallel to* $\vec{a} \times \vec{b}$
(B) $\vec{c}$ *lies in the plane containing* $\vec{a}$ *and* $\vec{b}$
(C) $\vec{a}, \vec{c} = 0$ *or* $\vec{b}.\vec{c}$
(D) $\vec{c}$ *is perpendicular to the plane containing* $\vec{a}$ *and* $\vec{b}$

83. If $X = [0,1]$ *and* $Y = [0,1] \cup [2,3]$ *and* $f : X \to Y$ is a map then
(A) f cannot be continuous
(B) f cannot be onto
(C) f cannot be $1-1$
(D) f cannot be monotone increasing

84. Mark the **wrong** statement
If X *is a metric space,* A *is subset of* X *and* $d(x,A) = \inf\{d(x,y) : y \in A\} \forall x \in X$,
(A) $d : X \to [0,\infty]$ *is continuous*
(B) *There is a sequence* $(x_n) \subseteq A$ *such that* $\lim_{n\to\infty} d(x, x_n) = d(x, A)$
(C) $d(x,A) = 0 \Leftrightarrow x \in \overline{A}$, *the closure of* A
(D) *There exists* $a \in A$ *such that* $d(x,a) = d(x,A)$

85. Let $\chi_E(x)=\begin{cases}1 & if\ x\in E\\ 0 & if\ x\in E^c\end{cases}$ If $f(x)=2\chi_{[0,1]}-3\chi_{\left[\frac{1}{2},1\right]}+\chi_{[1,2]}\ then \int_0^2 f(x)dx\ is$

(A) $\frac{2}{3}$ (B) $\frac{3}{2}$ (C) 0 (D) $\frac{5}{3}$

86. If $F(x)=\int_0^x \frac{x^{20}(\sin\frac{\pi}{2}x+\cos\frac{\pi}{2}x)}{1+x^2}dx\ for\ x\in[0,2]\ then\ F'(1)\ is$

(A) 0 (B) 1 (C) 2 (D) $\frac{1}{2}$

87. If $a_n=(3-\frac{1}{n+2})\cos n\pi,\ n\in\mathbb{N}\ then$

(A) $\limsup_n a_n=\liminf_n a_n$ (B) $\limsup_n a_n=\frac{7}{2}$

(C) $\liminf_n a_n=\frac{5}{2}$ (D) $\limsup_n a_n=3\neq\liminf_n a_n$

88. If $A=\begin{bmatrix}1 & 2\\ 1 & 2\end{bmatrix}\ then$

(A) A has no eigenvalue (B) 3 is the only eigenvalue of A

(C) 1 is the only eigenvalue of A (D) 0 and 3 are eigenvalue of A

89. If the only ideals of ring R are {0} and R then

(A) R is a field

(B) R is an integral domain

(C) R is a field provided R is commutative with unit element

(D) R is a commutative ring with unit element

90. If A is a $m\times n$ matrix and B is a matrix such that $AB=I$, where I is the identity matrix. Then

(A) B must be a $m\times n$ matrix (B) $m=n$

(C) B must be a $n\times m$ matrix (D) $m<n$

91. If $f(x)=x-\log_e x\ then$

(A) f *is monotone increasing on* $(0,\infty)$ (B) f *is monotone decreasing on* $(0,\infty)$

(C) $f\le 0$ *on* $(0,1)$ (D) $f>0$ *on* $(0,1)$

92. If $f(x)=5x^5-4x^4+3x^2-6x+1\ then$

(A) f *has a zero in* $(0,1)$ (B) $f(-1)<0$

(C) f *has 25 zeroes on* $\mathbb{R}$ (D) f *is not differentiable on* $\mathbb{R}$

93. $\lim_{x\to\infty}\frac{x^3+x^2+1}{x^{2x}}$

(A) ∞ (B) 0 (C) 1 (D) $\frac{1}{e}$

94. If $A=\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}$ and $V=\{AX : X\in\mathbb{R}^2\}$ then

(A) V is one dimensional vector space (B) V is two dimensional vector space

(C) V is not a vector space (D) $V=\{0\}$

95. The value of $\frac{1}{2\pi i}\int_c \frac{3z+1}{z(z-1)^3}dz$ where c is the circle $|z|=2$ is

(A) 0 (B) $\frac{1}{2\pi i}$ (C) $2\pi i$ (D) πi

96. The value of $\int_c \bar{z}dz$, where c is the line from $z=0$ to $z=1+i$ is

(A) 0 (B) 1 (C) $2\pi i$ (D) πi

97. If (X,d) is a metric space, $A\subseteq X$ and $x\in\bar{A}$ then

(A) x is not in closure of A^c

(B) x is in A

(C) For any $\in>0$ there is a y in A such that $d(x,y)<\in$

(D) There is a y in A such that $d(x,y)>1$

98. If $f:[a,b]\to\mathbb{R}$ is a bounded Riemann integrable function and $\int_a^b f(x)dx=0$ then

(A) $f\equiv 0$ on $[a,b]$

(B) $f\equiv 0$ on $[a,b]$ if f is continuous

(C) $f\equiv 0$ on $[a,b]$ if $f(x)\geq 0\ \forall\ x\in[a,b]$

(D) f can be non zero over an interval of positive length

99. The Euler equation $x^2y''+xy'+y=0$ can be transformed into an equation with constant coefficients by the transformation $z=$

(A) $\log x$ (B) e^x (C) x^2 (D) x

100. $y=x^m$ is not a solution of the equation $x^3y^{(4)}+8x^2y'''+8xy''-8y'=0$ when $m=$

(A) 2 (B) 0 (C) 1 (D) –1
