ENTRANCE EXAMINATION FOR ADMISSION, MAY 2010.

Ph.D. (STATISTICS)

COURSE CODE : 149

Register Number :

Signature of the Invigilator (with date)

COURSE CODE : 149

Time : 2 Hours

Max : 400 Marks

Instructions to Candidates :

1. Write your Register Number within the box provided on the top of this page and fill in the page 1 of the answer sheet using pen.
2. Do not write your name anywhere in this booklet or answer sheet. Violation of this entails disqualification.
3. Read each question carefully and shade the relevant answer (A) or (B) or (C) or (D) in the relevant box of the ANSWER SHEET using HB pencil.
4. Avoid blind guessing. A wrong answer will fetch you –1 mark and the correct answer will fetch 4 marks.
5. Do not write anything in the question paper. Use the white sheets attached at the end for rough works.
6. Do not open the question paper until the start signal is given.
7. Do not attempt to answer after stop signal is given. Any such attempt will disqualify your candidature.
8. On stop signal, keep the question paper and the answer sheet on your table and wait for the invigilator to collect them.
9. Use of Calculators, Tables, etc. are prohibited.

1. The basic feasible solution of a Linear Programming Problem (LPP) with m-constraints has

(A) exactly m basic a variables (B) more than m basic variables

(C) less than m basic variables (D) at the most m basic variables

2. The optimality test is applicable to a feasible solution of the transportation problem of order $m \times n$ if the number of allocations in independent position is equal to

(A) m+n-1 (B) m+n (C) (m-1)(n-1) (D) mn-1

3. In a zero-sum two-person game with saddle point, the players use only

(A) mixed Strategies (B) pure Strategies

(C) both Mixed and Pure Strategies (D) other than these two

4. In the post-optimality analysis of LPP, the feasibility condition is affected due to

(A) changes in the cost vector

(B) changes in the requirement vector

(C) addition and deletion of linear constraints

(D) both (B) and (C)

5. The time period over which the inventory level will be controlled is called the

(A) Re-order point (B) Re-order quantity

(C) Safety time (D) Time Horizon

6. The method used for solving an assignment problem is

(A) MODI method (B) Reduced Matrix method

(C) Hungarian method (D) Stepping Stone method

7. Re-order level of an item is always

(A) less than its minimum stock (B) more than its minimum stock

(C) more than its maximum stock (D) less than its maximum stock

8. When there are more than one server, customer behaviour in which he moves from one queue to another is

(A) Balking (B) Jockeying

(C) Reneging (D) Alternating

9. If there are n workers and n jobs, there would be

(A) n solutions (B) $n!$ solutions

(C) $(n-1)!$ solutions (D) $(n!)^n$ solutions

10. Under group replacement policy

(A) Group as well as individual replacements are done

(B) All items are replaced, irrespective of the fact that items have failed or have not failed

(C) The optimal group replacement interval is determined at the point where the sum of group replacement per unit of time and the cost of individual replacement is maximum

(D) All the above

11. If the p-component vector Y is distributed according to $N(0,\Sigma)$ and Σ is non-singular then $Y\Sigma^{-1}Y$ is distributed according to

(A) χ^2 distribution with p degrees of freedom

(B) χ^2 distribution with (p-1) degrees of freedom

(C) $N_p(0, Y\Sigma Y)$

(D) $N_p(0, Y\Sigma^{-1}Y)$

12. The relation between Hotelling T^2 statistic and Mahalanobis D^2 statistic for testing equality of two mean vectors of normal population is

(A) $T^2 = \left(\frac{N_1+N_2}{N_1N_2}\right)D^2$ (B) $T^2 = \left(\frac{N_1N_2}{N_1+N_2}\right)D^2$

(C) $T^2 = \left(\frac{N_1+N_2-2}{N_1N_2}\right)D^2$ (D) $T^2 = \left(\frac{N_1N_2}{N_1+N_2-2}\right)D^2$

13. Let the covariance matrix be $\Sigma = \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}$ then the variances of the two principal components are given by

(A) ρ, ρ^2 (B) $(1+\rho), \rho$

(C) $(1-\rho), \rho$ (D) $(1+\rho), (1-\rho)$

14. Consider the multiple regression model $Y = X\beta + \varepsilon$ where Y_{nx1} , $X_{nx(p+1)}$, $\beta_{(p+1)x1}$ and ε_{nx1}, $\varepsilon \sim N(0,\sigma^2 I)$. The m.l.e of β, $\hat{\beta} = (X'X)^{-1}X'Y$ is distributed according to

(A) $N(\beta,\sigma^2 I)$

(B) $N(\beta, I)$

(C) $N(\beta,\sigma^2(X'X)^{-1})$

(D) $N(\beta,\sigma^2(X'X))$

15. A measure function is a

(A) point function on Master set with Range$(-\infty,\infty)$

(B) point function on a σ - field with Range$(0,\infty)$

(C) set function on Master set with Range$(0,\infty)$

(D) set function on a σ - field with Range$(0,\infty)$

16. The sum of two simple functions is

(A) a measurable set function

(B) always a simple function

(C) never a simple function

(D) sometimes a simple function

17. The connection between almost sure (a.s) convergence, convergence in probability (p) and in mean (m) is

(A) a.s $\Rightarrow$ m $\Rightarrow$ p

(B) a.s $\Rightarrow$ p; p $\Rightarrow$ m

(C) a.s $\Rightarrow$ p; m $\Rightarrow$ p

(D) m $\Rightarrow$ a.s $\Rightarrow$ p

18. The inequality $E^{1/r}(|X|+|Y|)^r \le E^{1/r}(|X|^r) + E^{1/r}(|Y|^r)$ is due to

(A) Schwartz

(B) Minkowski

(C) Cauchy

(D) Rao-Blackwell

19. If X^2 and Y^2 are independent then

(A) X and Y are independent

(B) X and Y need not be independent

(C) $E(X^2) = E(Y^2)$

(D) $E(XY) = 0$

20. If X_n is a sequence of iid random variables then the result that $\frac{S_n}{n} \xrightarrow{p} \mu \ (\mu = E(X_k))$ is the consequence of

(A) S.L.L.N (B) W.L.L.N

(C) Daniel Kolmogrov theorem (D) Continuity theorem

21. A random sample $X_1, X_2, \ldots X_n$ is drawn from a distribution with density function

$$f(x,\theta) = \theta(1-x)^{1-\theta}; 0 < x < 1$$
$$\theta > 0$$

A sufficient statistics for θ is

(A) $\sum X_i$ (B) Sample median

(C) $\sum \log(1-X_i)$ (D) $\sum X_i^2$

22. Let $(X_1, X_2, \ldots X_n)$ be a random sample of observations with mean μ and finite variance. Then for estimating μ, the statistic $T_n = 2\sum_{i=1}^{n} i X_i / n(n+1)$ is

(A) unbiased and consistent (B) biased and consistent

(C) unbiased but not consistent (D) biased and not consistent

23. X_1, X_2, X_3 are independent observations from a normal population with mean θ and variance unity.

Two statistics T_1 and T_2 are defined as $T_1 = \frac{X_1 + X_2 + 2X_3}{4}$ and $T_2 = \frac{X_1 + X_2 + X_3}{3}$. The efficiency of T_1 relative to T_2 is given by

(A) $\frac{8}{9}$ (B) $\frac{3}{4}$ (C) $\frac{9}{8}$ (D) $\frac{4}{3}$

24. Three random observations X_1, X_2, X_3 are made on a Poisson distribution with parameter λ.

Let $g(\lambda) = e^{-\lambda}$. Then the Cramer Rao lower bound for the variance of unbiased estimators of $g(\lambda)$ is given by

(A) $\frac{\lambda e^{-2\lambda}}{3}$ (B) $e^{-\lambda/3}$ (C) $\lambda^2 e^{-\lambda}$ (D) $e^{-3\lambda}$

25. Let $X_1, X_2, \ldots X_n$ be a random sample from a distribution with density

$$f(x;\theta)=\begin{cases}\theta . e^{-\theta x}; x>0; \theta>0\\ 0; otherwise\end{cases}$$

The moment estimator of θ is

(A) $\sum X_i/n$ (B) Sample median

(C) $n/\sum X_i$ (D) $(\sum X_i)^{\frac{1}{n}}$

26. Let X have the density function $f(x,\theta)=\frac{3x^2}{\theta^3} for 0<x<\theta$

$=0$ otherwise

If $P(X>1) = 7/8$, what is the value of θ?

(A) $\frac{1}{2}$ (B) 2 (C) $2^{1/3}$ (D) $(7/8)^{1/3}$

27. If A and B are two events such that $P(A)= P(B)=1/3$ and $P(A/B) =1/6$, what is $P(A' \cap B')$?

(A) 7/18 (B) 1/6 (C) 1/3 (D) ½

28. Let X and Y be two independent uniform random variable over the interval [100,200] What is the number t for which the probability that atleast one of X and Y exceeds t is ¼ ?

(A) 150 (B) $200-50\sqrt{3}$ (C) $100+50\sqrt{3}$ (D) 175

29. Suppose X is continuous random variable with Uniform distribution having mean 1 and variance 4/3. What is $P(X<0)$?

(A) 0 (B) ¼ (C) 1/12 (D) none of these

30. Let X_1 and X_2 be two independent standard normal variables. Let $Y=\frac{(X_2-X_1)^2}{2}$.

Then $V(Y)$ is

(A) 1 (B) 2 (C) 1/2 (D) none of these

31. In a stratified random sampling scheme the population characteristics can be more efficiently estimated from a stratified sample than from overall simple random sample if

(A) Strata means differ widely, and within strata variation is high

(B) Strata means do not differ widely, and within strata variation is high

(C) Strata means differ widely, and within strata variation is low

(D) Strata means do not differ widely, and within strata variation is low

32. An unbiased estimate of variance of the sample proportion p is

(A) $\left(\frac{N-n}{N}\right)\frac{p(1-p)}{n}$ (B) $\left(\frac{N-n}{N}\right)\frac{p(1-p)}{n-1}$

(C) $\left(\frac{N-n}{N-1}\right)\frac{p(1-p)}{n}$ (D) $\left(\frac{N-n}{N-1}\right)\frac{p(1-p)}{n-1}$

33. The Name "Two- Stage Sampling is due to-

(A) Cochran (B) Fisher

(C) Midzuno (D) Mahalanobis

34. If two Latin squares of the same order, but with different set of symbols are superimposed one over the other and each symbol of one falls on each symbol of the other once and only once, then the two Latin squares are said to be

(A) Independent (B) Identical

(C) Orthogonal (D) None of these

35. If the number of levels of each factor in an experiment is different then the experiment is called as

(A) Factorial (B) Asymmetrical

(C) Incomplete (D) None of these

36. The analysis of experiments with missing observation avoiding the complication of analysis of non orthogonal data was initialized by

(A) R.A. Fisher (B) F. Yates

(C) O. Kempthorne (D) R.C. Bose

37. Models in which some factors are fixed and some are random are called as

(A) Fixed models
(B) Random models
(C) Growth models
(D) Mixed models

38. A random sample of size 1 say X_1 is taken from $N(\mu,\sigma^2)$. Then the MLE of (μ,σ^2) is

(A) X_1
(B) (sample mean, sample variance)
(C) (μ,σ^2)
(D) does not exist.

39. Let $X_1, X_2,\ldots\ldots, X_n$ be a random sample from $U(0,\theta)$. Then an unbiased estimator for θ is

(A) $\sum x_i/n$
(B) $(n+1)/n\ Max(x_1,\ldots x_n)$
(C) $((n+1)/n)\ Min(x_1,\ldots x_n)$
(D) $(Max(x_1,\ldots x_n) - Min(x_1,\ldots x_n))$

40. Let $X_1, X_2,\ldots\ldots, X_n$ be iid random variables. $X_i \sim N(\mu,\sigma^2)$. Then to find the $(1-\alpha)$ level confidence interval for σ^2 when μ is known we use

(A) t distribution
(B) χ^2 distribution
(C) F distribution
(D) Normal distribution

41. Let $X \sim N(\mu,\sigma^2)$. Suppose both μ and σ are unknown. Then which one of the following statement is incorrect?

(A) H: $\mu \le \mu_0$, $\sigma^2 > 0$ (μ_0 is known constant) is a composite hypothesis
(B) H: $\mu > \mu_0$, $\sigma^2 > 0$ (μ_0 is known constant) is a composite hypothesis
(C) H: $\mu = \mu_0$, $\sigma^2 > 0$ is a composite hypothesis
(D) H: $\mu = \mu_0$, $\sigma^2 = 0$ is a composite hypothesis

42. The level of significance of a test is

(A) Probability of rejecting the null hypothesis when it is true
(B) Probability of rejecting the alternate hypothesis when it is true
(C) Specified upper bound for the probability of rejecting the null hypothesis when it is true
(D) None of the above

43. Let ϕ be a test for testing H: $\theta \; \varepsilon \; \theta_0$ against K: $\theta \; \varepsilon \; \theta_1$. Then the size of the test of

(A) $E_\theta \phi(x); \theta \; \varepsilon \; \theta_0$

(B) $\sup_{\theta \varepsilon \theta_0} E_\theta \phi(x)$

(C) $\sup_{\theta \varepsilon \theta_1} E_\theta \phi(x)$

(D) $\inf_{\theta \varepsilon \theta_0} E_\theta \phi(x)$

44. Match List 1 with List 2 and select the answer using the codes given below the list.

List 1	List 2
a. Lehman-Scheffe theorem	1. To obtain a sufficient Statistic
b. Factorization theorem	2. To obtain sharper lower bounds for variance of unbiased estimation.
c. Cramer Rao inequality	3. To obtain UMVUE.
d. Bhattacharya Inequality	4. Lower bound for variance of an unbiased estimator.

The correct match is

	a	b	c	d
(A)	3	4	1	2
(B)	3	1	4	2
(C)	1	2	3	4
(D)	4	2	3	1

45. Neyman Pearson Lemma helps to obtain the most powerful test for testing

(A) a simple hypothesis against a simple alternative.

(B) a simple hypothesis against a composite alternative.

(C) a composite hypothesis against a composite alternative

(D) a composite hypothesis against a simple hypothesis.

46. Let $X_1, \ldots, X_n$ be a random sample from $N(\mu, \sigma^2)$ where both μ and σ^2 are unknown. For testing H: $\mu \leq \mu_0$; $\sigma^2 > 0$ against K: $\mu > \mu_0$; $\sigma^2 > 0$, we use

(A) Two tailed t-test

(B) Two tailed χ^2 test

(C) One tailed t-test

(D) One tailed χ^2 test

47. Assertion (A): We speak of consistency of a sequence of estimates rather than one point estimate.

Reason (R): Consistency is essentially a large sample property.

Choose the correct answer.

(A) Both A and R are true and R is the correct explanation of A.

(B) Both A and R are true but R is not the correct explanation for A.

(C) A is true but R is false.

(D) A is false but R is true.

48. Given that $P(A_1 \cup A_2) = 5/6$, $P(A_1 \cap A_2) = 1/3$, $P(A_2) = 1/2$, the events are

(A) mutually exclusive (B) independent

(C) dependent (D) equally likely

49. Let X be a Beta distributed random variable with parameters α and β. The expectation of X is

(A) α / β (B) $\alpha\ (\alpha + \beta)$ (C) $\alpha\ \beta$ (D) $(\alpha - \beta)$

50. In testing of hypothesis problems

(A) Probability of Type I error is fixed and probability of type II error is minimized.

(B) Probability of Type II error is fixed and probability of type I error is minimized.

(C) The probabilities of both errors are fixed.

(D) Both of them are not fixed and they are minimized simultaneously.

51. Match list 1 and list 2 and select the correct answer using the codes given below the list.

List 1	List 2
1. Randomised test	a. Probability of rejecting the null hypothesis when the alternative hypothesis is true.
2. Non Randomised test	b. $0 \le \phi(x) \le 1$
3. Level of the test	c. Probability of rejecting the null hypothesis when it is true.
4. Power of the test	d. $\phi(x) = 0$ or 1
5. Size of the test	

The correct match is

	a	b	c	d
(A)	4	1	5	2
(B)	4	2	1	3
(C)	5	3	2	1
(D)	2	1	3	5

52. Let Ω be the parameter space. A statistic T is said to be unbiased for $g(\theta)$.

(A) $E_{\theta}(T) = g(\theta) \quad \forall \theta \in \Omega$

(B) $E_{\theta}(T) = g(\theta)$ for atleast one $\theta \in \Omega$

(C) $V_{\theta}(T) = g(\theta) \quad \forall \theta \in \Omega$

(D) $V_{\theta}(T) = g(\theta)$ for atleast one $\theta \in \Omega$

53. In two stage cluster sampling the variance of an estimator $\hat{\theta}$ is given by

(A) $E_1(V_2(\hat{\theta})) + V_1(E_2(\hat{\theta}))$

(B) $E_1(V_1(\hat{\theta})) + E_1(V_2(\hat{\theta}))$

(C) $E_2(V_1(\hat{\theta})) + V_2(E_1(\hat{\theta}))$

(D) $E_1(V_2(\hat{\theta})) + E_2(V_1(\hat{\theta}))$

54. Given two random samples, one from each of two populations with the same unknown variance and unequal means, the likelihood ratio criterion for testing of the hypothesis of equal means leads to

(A) Chi-square test

(B) Normal test

(C) t-test

(D) F-test

55. If a distribution possesses the monotone likelihood ratio property, then

(A) for every randomized test, there exists an equally good non randomized test.

(B) there exists a uniformly most powerful test for one-sided alternatives for testing composite hypotheses.

(C) the likelihood ratio test statistic is distributed normally.

(D) expectation of the likelihood ratio test statistic is equal to zero.

56. If the sampling fraction is the same in all strata; then the allocation is called

(A) Equal Allocation

(B) Proportional Allocation

(C) Optimum Allocation

(D) Random Allocation

57. If $E(X^3)$ exists then

(A) $E(X^r)$; $0<r<3$ exists

(B) $E(X^{3+\partial})$ exists for $\partial>0$

(C) The distribution of X is symmetric

(D) $E|X|^3$ need not exist

58. If $X_n \xrightarrow{a.s} X$, then

(A) $P(\lim X_n = X) = 0$

(B) $P(\lim X_n \neq X) = a; 0 < a < 1$

(C) $P(\lim X_n = X) = 1$

(D) $P(\lim X_n \neq X) = 1$

59. For a degenerate random variable at $a \in R$

(A) $P(X = a) = 0$

(B) $P(X = a) = \infty$

(C) $P(X = a) = -\infty$

(D) $P(X = a) = 1$

60. X_1, X_2 are two independent observations on X having the distribution $P(X=0)=\theta=1-P(X=1)$ then the statistic X_1-X_2 is

(A) sufficient and complete

(B) not sufficient but complete

(C) not complete but sufficient

(D) not sufficient and not complete

61. A random sample of two observations X_1, X_2 is drawn from Bernoulli distribution with probability mass function

$$f(x;\theta)=\begin{cases}\theta^x.(1-\theta)^{1-x}; x=0,1\\ 0; otherwise\end{cases}$$

where $\theta \in \left[\frac{1}{4},\frac{3}{4}\right]$. The M.L.E of θ is given by

(A) $(X_1 + X_2)/2$

(B) $(X_1 + X_2 + 1)/4$

(C) $(2X_1 + X_2)/3$

(D) $(X_1 - X_2)/2$

62. If $X_1, X_2, \ldots X_n$ are i.i.d $N(\theta, \sigma^2)$, σ^2 is unknown. Then the likelihood ratio test for testing $H_0: \theta = \theta_0$ against $H_1: \theta \neq \theta_0$ leads to

(A) one sided χ^2 test

(B) one sided student's t test

(C) two sided student's t test

(D) two sided χ^2 test

63. A rectangle is to be constructed having dimensions X and 2X where X is a random variable with density function $f(x) = x/2; \quad 0 < x < 2$

$= 0 \quad$ otherwise

What is the expected area of the rectangle?

(A) 2 (B) 4 (C) 8 (D) 16

64. Identify the generalized interaction in 2^5 factorial of block 2^3 when the independent interactions are ABC and ADE

(A) ABDE (B) ACDE (C) ABCD (D) BCDE

65. Partial confounding is defined as

(A) the same set of treatments are confounded in all replications

(B) different treatments are confounded in different replications

(C) some treatments are confounded and some or not

(D) none of the above

66. The variance of estimate of difference between two treatment effects when there are two missing observations on LSD is given by

(A) $\frac{2\sigma^2}{r}\frac{r-2}{r-3}$

(B) $\frac{2\sigma^2}{r}$

(C) $\frac{2\sigma^2(r-2)}{r}$

(D) $\frac{2\sigma^2}{r(r-3)}$

67. Let $X_1, X_2, \ldots X_n$ be a i.i.d random variables from Bernoulli distribution with parameter θ. The U.M.V.U.E of $g(\theta)=\theta\,(1-\theta)$ in terms of $T=\sum X_i$ is

(A) $T^2/(n-1)$

(B) $(nT-T^2)/n(n-1)$

(C) $(T(1-T))/n$

(D) $(T^2-T)/n^2$

68. The number of independent contrasts on which a sum of square is based is called

(A) Multiple contrast

(B) Simple contrast

(C) Mean Squares

(D) Degrees of freedom

69. For a two-person zero-sum game with two players A and B and payoff matrix for player A, the optimum strategies are

(A) Minimax for A and Maximin for B

(B) Maximax for A and Minimax for B

(C) Minimin for A and Maximin for B

(D) Maximin for A and Minimax for B

70. For any primal and its dual

(A) Optimum value of the objective function is same

(B) Both primal and dual cannot be feasible

(C) Primal will have an optimum solution if and only if dual does to

(D) All the above

Answer any **THREE** questions. ($3 \times 10 = 30$ marks)

71. When is a matrix said to be idempotent? Examine the nature of the eigen values of an idempotent matrix.

72. Define the maximum likelihood estimator (m.l.e). State its properties. Also prove any one asymptotic property of the m.l.e.

73. Write down the model for the Randomized Block Design along with the assumptions. Also explain the analysis of data arising from such a design.

74. Define convergence in probability and almost sure convergence. State sufficient conditions for the same and illustrate with an example.

75. What are non-sampling errors? Explain the necessity and the methodology of controlling them.

76. Describe the Likelihood Ratio Test (LRT). Is it consistent? Also obtain the LRT for testing $\mu \leq \mu_0$ against $\mu > \mu_0$ using a random sample from $N(\mu, \sigma^2)$, σ^2 known.