COURSE CODE : 375

Time : 2 Hours

Max : 400 Marks

Instructions to Candidates :

1. Write your Register Number within the box provided on the top of this page and fill in the page 1 of the answer sheet using pen.

2. Do not write your name anywhere in this booklet or answer sheet. Violation of this entails disqualification.

3. Read each question carefully and shade the relevant answer (A) or (B) or (C) or (D) in the relevant box of the ANSWER SHEET using HB pencil.

4. Avoid blind guessing. A wrong answer will fetch you –1 mark and the correct answer will fetch 4 marks.

5. Do not write anything in the question paper. Use the white sheets attached at the end for rough works.

6. Do not open the question paper until the start signal is given.

7. Do not attempt to answer after stop signal is given. Any such attempt will disqualify your candidature.

8. On stop signal, keep the question paper and the answer sheet on your table and wait for the invigilator to collect them.

9. Use of Calculators, Tables, etc. are prohibited.
1. A sample study of the people of an area revealed that the total number of women was 40% and the percentage of coffee drinkers was 45 and the percentage of male coffee drinkers was 20. What is the percentage of female non-coffee drinkers?
   (A) 10 (B) 15 (C) 18 (D) 20

2. The arithmetic mean and geometric mean of two observations are 5 and 4 respectively. Then the observations are
   (A) 2, 8 (B) 4, 1 (C) 6, 4 (D) 3, 7

3. What is the harmonic mean of 1, 1/2, 1/3, ...., 1/n ?
   (A) n (B) 2n (C) 2/(n + 1) (D) n (n+1)/2

4. If the arithmetic mean and geometric mean of 10 observations are both 15, then the value of harmonic mean is
   (A) less than 15 (B) 15 (C) more than 15 (D) cannot be determined

5. If arithmetic mean and coefficient of variation in percentage of x are 20 and 20 respectively, then what is the variance of \( y = 10 - 2x \)?
   (A) 64 (B) 16 (C) 36 (D) 84

6. If the relationship between two variables x and y is given by \( 2x + 3y + 4 = 0 \), then the value of the correlation coefficient between x and y is
   (A) 1 (B) -1 (C) 0 (D) cannot be determined

7. If the coefficient of correlation between two variables is -0.4, then the coefficient of determination is
   (A) 0.84 (B) 0.6 (C) 0.16 (D) -0.6

8. The arithmetic means of x and y are 80 and 98 respectively and the variances of x and y are 4 and 9 respectively. If the value of the correlation coefficient between x and y is obtained as 0.6, then what is the most likely value of y when x = 90?
   (A) 90 (B) 103 (C) 104 (D) 107

9. For a bivariate set of 5 observations, if the sum of squares of the differences in ranks is obtained as 24, then the value of rank correlation is
   (A) 0.2 (B) -0.4 (C) 0.40 (D) -0.2
10. If the two lines of regression are obtained as $2x + 3y = 6$ and $2x + 4y = 2$, then the arithmetic mean of $x$ and $y$ are
   (A) $9, -4$  \hspace{1cm} (B) $-9, 4$  \hspace{1cm} (C) $4, 9$  \hspace{1cm} (D) $-4, 9$

11. If the regression coefficients of $x$ on $y$ and $y$ on $x$ are respectively $-1$ and $-0.25$, the correlation coefficient between $x$ and $y$ is
   (A) $0.5$  \hspace{1cm} (B) $-0.5$  \hspace{1cm} (C) $0$  \hspace{1cm} (D) $-1$

12. If the regression lines of $y$ on $x$ and $x$ on $y$ are identical, then the correlation coefficient between $x$ and $y$ is
   (A) $+1$  \hspace{1cm} (B) $-1$  \hspace{1cm} (C) $\pm 1$  \hspace{1cm} (D) $0$

13. Mr. $X$ is appearing for English and Accountancy examination. The probability of passing English and Accountancy examinations are $1/2$ and $1/3$ respectively. What is the probability that he will pass in both examinations?
   (A) $1/5$  \hspace{1cm} (B) $1/2$  \hspace{1cm} (C) $1/6$  \hspace{1cm} (D) $5/6$

14. A box contains 5 red and 4 white marbles. Two marbles are drawn successively from the box without replacement and it is noted that the second one is white. What is the probability that the first is also white?
   (A) $1/8$  \hspace{1cm} (B) $3/8$  \hspace{1cm} (C) $5/8$  \hspace{1cm} (D) $7/8$

15. A continuous random variable has the following p.d.f., $f(x) = 3x^2$; $0 \leq x \leq 1$
   If $P(X \leq a) = P(X > a)$, then the value of $a^3$ is
   (A) $1/4$  \hspace{1cm} (B) $1/16$  \hspace{1cm} (C) $1/2$  \hspace{1cm} (D) $1/8$

16. Let $X_1 \sim N(2, 1)$ and $X_2 \sim N(3, 2)$, then the distribution of $2X_1 + 3X_2$ is
   (A) $N(12, 15)$  \hspace{1cm} (B) $N(15, 12)$  \hspace{1cm} (C) $N(22, 13)$  \hspace{1cm} (D) $N(13, 22)$

17. Let $X$ follow uniform distribution over the interval $(2, 4)$. Then the mean and variance are
   (A) $3, 1/3$  \hspace{1cm} (B) $2/3, 4$  \hspace{1cm} (C) $1/3, 6$  \hspace{1cm} (D) $1/3, 2$

18. A pair of distributions satisfying memory less property is
   (A) Exponential and Gamma  \hspace{1cm} (B) Geometric and Chi-square
   (C) Exponential and Geometric  \hspace{1cm} (D) Exponential and Normal
19. If a random variable \( X \) has the Probability density function as follows:
\[
f(x) = \begin{cases} 
\frac{1}{4} & -2 < X < 2 \\
0 & \text{otherwise}
\end{cases}
\]
and zero otherwise. Find \( P(X < 1) \)

(A) 2/3  (B) 3/4  (C) 1/4  (D) 1/2

20. The distribution corresponding to the Moment Generating Function \( e^t(5 - 4e^t)^{-1} \) is

(A) Poisson  (B) Geometric  (C) Binomial  (D) Hyper geometric

21. For a Binomial distribution with \( n = 10 \) and \( p = 1/2 \), the mode of the distribution is at

(A) \( x = 2 \)  (B) \( x = 3 \)  (C) \( x = 4 \)  (D) \( x = 5 \)

22. The p.d.f. of a r.v. \( X \) is \( f(x) = 2e^{-2x}, x > 0 \). Then \( F(2) \) is

(A) \( \frac{e^4 - 1}{e^4} \)  (B) \( \frac{e - 1}{e} \)  (C) \( \frac{e^3 - 1}{e} \)  (D) \( \frac{e + 1}{e - 1} \)

23. The mgf of Normal distribution with mean 2 and variance 1 is equal to

(A) \( e^{2t + t^2 / 2} \)  (B) \( e^{-2t + t^2 / 2} \)  (C) \( e^{3t / 2} \)  (D) \( e^{2(t + t^2)} \)

24. \( X \) and \( Y \) are independent with common Exponential distribution with parameter \( \theta = 1 \), then the distribution of \( X - Y \) is

(A) A Standard Cauchy distribution  (B) An Exponential distribution  
(C) A Standard Laplace distribution  (D) A Standard Normal distribution

25. Let \( X \) and \( Y \) be two independent Binomial random variables with parameters \( (2, 1/3) \) and \( (7, 1/3) \). Then \( P[X + Y = 3] \) is equal to

(A) \( \binom{9}{3} \left( \frac{1}{3} \right)^3 \left( \frac{2}{3} \right)^6 \)  (B) \( \binom{9}{3} \left( \frac{2}{3} \right)^3 \left( \frac{1}{3} \right)^6 \)

(C) \( \binom{7}{2} \left( \frac{1}{3} \right)^2 \left( \frac{2}{3} \right)^5 \)  (D) \( \binom{9}{3} \left( \frac{1}{3} \right)^3 \left( \frac{2}{3} \right)^8 \)
26. Let \( (X,Y) \) follow Bivariate Normal with \( E(X) = 2, E(Y) = 3, r(X,Y) = 0.6 \), Var \( X = 4 \), Var \( Y = 2 \). Then the conditional mean of \( X \) given \( Y = y \) is

(A) \( 0.8y - 0.4 \)  
(B) \( 0.4y - 0.8 \)  
(C) \( 0.2y - 0.4 \)  
(D) \( 0.4y - 0.2 \)

27. The m.g.f. of \( X \) is given by \( M_X(t) = 3/(3-t) \). Then the mean and variance of \( X \) are

(A) \( 1/3, 2/9 \)  
(B) \( 1/3, 1/9 \)  
(C) \( 1/9, 1/3 \)  
(D) \( 1/2, 1/4 \)

28. The r.v. \( X \) has Poisson distribution such that \( P(X = 1) = P(X = 2) \) then \( P(X = 0) \) is

(A) \( e^{-3} \)  
(B) \( e^{-5} \)  
(C) \( e^{-2} \)  
(D) \( e^{-1} \)

29. If \( X \) follows chi-square distribution with mean 2 and \( Y \) follows chi-square distribution with mean 1 and is distributed independently of \( X \), then the distribution of \( X/(X+Y) \) is

(A) \( B_1(2,1) \)  
(B) \( B_1(1,1/2) \)  
(C) \( B_1(1/2,1) \)  
(D) \( B_2(1,1/2) \)

where \( B_i \) stands for Beta distribution of type \( i = 1, 2 \).

30. \( X \) is a continuous random variable (r.v) with probability density function \( f(x) \) and distribution function \( F(x) \). Then \( P(X = a) \), where \( a \) is a given real number, is

(A) 0  
(B) \( f(a) \)  
(C) \( F(a) \)  
(D) \( f(a + 0) - f(a - 0) \)

31. \( X \) is a discrete r.v. with \( P(X = j) = 1/2^j, j = 1,2,3, \ldots \) What is the probability that a random observation on \( X \) is an even number?

(A) \( 1/3 \)  
(B) \( 1/2 \)  
(C) \( 1/4 \)  
(D) \( 1/8 \)

32. Quality aims at

(A) Measured dimensions are within the specifications

(B) Acceptance of all the items produced

(C) Consistent performance for the longer period

(D) High productivity

33. Statistical Process Control mainly deal with

(A) Control charts  
(B) Process capability analysis

(C) Pareto's charts  
(D) Sampling Plans
34. Control charts are used to
   (A) Identify the presence of chance causes
   (B) Identify the trend of the process
   (C) Identify the presence of assignable causes
   (D) Identify the tool consumption rate

35. A process is not under the state of statistical control implies
   (A) Exactly one point falls outside the control limits
   (B) More than one point falls outside the control limits
   (C) Presence of trend with seven or more points
   (D) At least one of the above cases is true

36. The process capability index $C_p$ is used to measure
   (A) The quality of the product  (B) The suitability of the process
   (C) The acceptance level of the product  (D) The rejection level of the product

37. A research report concludes that there are significant differences among treatments, with $F(2, 27) = 8.62, p < .01$. How many treatment conditions were compared in this study?
   (A) 2  (B) 3  (C) 29  (D) 30

38. In a RBD the equality of treatments effects is tested with the $F$ ratio with df (4, 20). The total number of observations considered in this design is
   (A) 24  (B) 29  (C) 30  (D) 32

39. A research report based on a Latin square design with some missing values provides the degrees of freedom for error mean square as 17. Then the number of missing values is
   (A) 4  (B) 3  (C) 2  (D) 5

40. The difference between the estimate and the parameter in a sample survey is known as
   (A) Non-Sampling Error  (B) Population Variance
   (C) Sampling Error  (D) Sampling Variance
41. The Variance of Systematic Sample mean of the population (Population size \(N\), Sample Size \(n\) and sampling interval \(k\) such that \(N = nk\)) with a linear trend is

(A) \(\frac{(N - 1)(n + 1)}{12}\)  \hspace{1cm} (B) \(\frac{(k - 1)(k + 1)}{12n}\)

(C) \(\frac{(k - 1)(k + 1)}{12}\)  \hspace{1cm} (D) \(\frac{(k - 1)(n + 1)}{12}\)

42. \(Q = \sum \frac{q_0 p_{o_j}}{\sum q_0 p_{o_j}} \times 100\) is the formula for

(A) Laspeyre’s price index number

(B) Laspeyre’s quantity index number

(C) Paasche’s price index number

(D) Paasche’s quantity index number

43. Which index number satisfies both time reversal test and factor reversal test?

(A) Fisher’s index number

(B) Marshall Edgeworth index number

(C) Walsh index number

(D) Kelly’s index number

44. The value of \(x\) in formula,

\[
\text{Chain index number} = \frac{\text{Current year LinkRelative} \times x}{100}
\]

(A) Preceeding year Chain index number

(B) Preceeding year link relative

(C) Succeeding year link relative

(D) Succeeding year Chain index number

45. Method of moving averages is used for measurement of

(A) Secular trend

(B) Seasonal trend

(C) Cyclic trend

(D) Irregular trend

46. \(X\) is a random variable taking values 1 and 2 with probabilities \(p\) and \(q\), \(p + q = 1\), to test \(H: p = 0.2\), a single observation is made on \(X\) (say \(x\)). A test rejects \(H\) if \(x = 1\). What is the size of the test?

(A) 0.8  \hspace{1cm} (B) less than 0.2

(C) greater than 0.2  \hspace{1cm} (D) 0.2
47. If \( T_1 \) is an MVUE of \( \gamma(\theta); \theta \in \Theta \) and \( T_2 \) is any other unbiased estimator of \( \gamma(\theta) \) with efficiency \( e_\theta \), the correlation coefficient between \( T_1 \) and \( T_2 \), say \( \rho_\theta \), equals

(A) \( e_\theta \)  
(B) \( e_\theta^2 \)  
(C) \( \frac{1}{\sqrt{e_\theta}} \)  
(D) \( \sqrt{e_\theta} \)

48. Every UMP critical region is necessarily

(A) Biased  
(B) A null set  
(C) An infinite set  
(D) Unbiased

49. The interval estimate for a single population mean when \( \sigma \) is known is given by

(A) \( x \pm z_\alpha \sigma \)  
(B) \( x \pm z_{\alpha/2} \sigma \)  
(C) \( x \pm z_\alpha \)  
(D) \( x \pm z_{\alpha/2} \)  

50. A two-tail statistical test is

(A) A statistical test for which the critical region comprises both large and small values of the test statistic.  
(B) When the alternative hypothesis is two sided.  
(C) A statistical test for which the critical region comprises small values of the test statistic.  
(D) A statistical test for which the critical region comprises large values of the test statistic.

51. If \( X \) is a Poisson variate with parameter \( \lambda \), then the unbiased estimator based on a single observation \( x \) of \( e^{-3\lambda} \) is

(A) \( (-3)^x \)  
(B) \( (-2)^x \)  
(C) \( 3^x \)  
(D) \( 2^x \)

52. The Maximum Likelihood Estimator of \( \theta \) in a random sample of size \( n \) from \( U(0, \theta) \) is

(A) The sample mean  
(B) The sample median  
(C) The largest order statistics  
(D) The smallest order statistics.

53. Let \( X_1, X_2, \ldots, X_n \) be a random sample from \( B(1, p) \), then the consistent estimator of \( p(1-p) \) is

(A) \( \bar{X} \)  
(B) \( \bar{X}^2 \)  
(C) \( \bar{X}(1-\bar{X}) \)  
(D) \( n \cdot \bar{X} \)
54. The power of a statistical test depends upon
   (i) sample size
   (ii) level of significance
   (iii) variance of sampled population
   (iv) the difference between the value specified by null and alternative hypothesis.
   (A) (i) and (ii)    (B) (ii) and (iii)    (C) (i) and (iv)    (D) all the four

55. A statistic \( T \) is said to be an unbiased estimator of \( \theta \) if, for all \( \theta \),
   (A) \( E(T) > \theta \)    (B) \( E(T) = \theta \)    (C) \( E(T) < \theta \)    (D) \( E(T) = 0 \)

56. For the following \( 2 \times 2 \) contingency table for two attributes the value of chi-square is

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<tr>
<td>B</td>
<td>20</td>
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<tr>
<td>b</td>
<td>10</td>
<td>40</td>
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(A) 100/21    (B) 10/18    (C) 10/38    (D) 20/36

57. Two dice are thrown and the sum of the numbers which come up on the dice is noted. The following events are considered
   (i) the sum is even
   (ii) the sum is multiple of 3
   (iii) the sum is less than 4
   (iv) the sum is greater than 11

The pair of events, which are mutually exclusive is
   (A) (i) and (ii)    (B) (ii) and (iii)    (C) (i) and (iv)    (D) (iii) and (iv)

58. Mr. X and Mr. Y appear in an interview for two vacancies in the same post. The probability of X's selection is \( \frac{1}{7} \) and that of Y's selection is \( \frac{1}{5} \). Then the probability that none of them will be selected is

(A) \( \frac{24}{35} \)    (B) \( \frac{23}{35} \)    (C) \( \frac{22}{35} \)    (D) \( \frac{21}{35} \)

59. The death rate obtained for a segment of a population is known as

(A) Specific death rate    (B) Crude death rate
(C) Standardized rate      (D) Vital index
60. The probability of a person aged $x$ living for $n$ more years is given by the formula

(A) $\frac{l_{x+n}}{l_x}$  (B) $\frac{(l_x - l_{x+n})}{l_x}$  (C) $\frac{(l_x - l_{x+n})}{l_{x+n}}$  (D) $\frac{l_x}{l_{x+n}}$

61. If $u = x^2 + xy + y^2$ then $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$ is

(A) $2u$  (B) $\frac{u}{2}$  (C) $u$  (D) $u + 2$

62. If $f(x) = \begin{cases} x, & x \leq 1 \\ ax, & 1 < x < 2 \\ x^2 + bx + 4, & x \geq 2 \end{cases}$ and $f(x)$ is differentiable everywhere then

(A) $a = 1$  $b = -2$  (B) $a = 3$  $b = -4$  (C) $a = 2$  $b = -3$  (D) $a = 1$  $b = -3$

63. If $y = \log (e^x+1)$, then $\frac{dy}{dx}$ is

(A) $\frac{e^x + 1}{e^x}$  (B) $\frac{1}{e^x + 1}$  (C) $\frac{e^x}{e^x + 1}$  (D) $e^x + 1$

64. If $y = f(x)$ is twice differentiable and has a minimum value at $x = a$, then

(A) $f''(a) < 0$  (B) $f''(a) > 0$

(C) $f''(a) = 0$  (D) $f''(a)$ is a constant

65. Let $f(x) = 3x - 1$ and $g(x) = x + 4$, then $\frac{df}{dg}$ is

(A) 2  (B) 1  (C) 0.4  (D) 3

66. If $u = y \sin x$ then $\frac{\partial^2 u}{\partial x \partial y}$ is equal to

(A) $\cos x$  (B) $\cos y$  (C) $\sin x$  (D) 0

67. If $y = a^x$ where $a$ is a constant then $\frac{dy}{dx}$ is equal to

(A) $\log a$  (B) $a^x \log a$  (C) $xa^{x-1}$  (D) $x \log a$
68. If \( x = at^3 \) and \( y = 2at \) where \( a \) is a constant then \( \frac{dy}{dx} \) is equal to

(A) \( at^2 \)  \hspace{1cm} (B) \( \frac{1}{at} \)  \hspace{1cm} (C) \( \frac{1}{t} \)  \hspace{1cm} (D) \( \frac{a}{t} \)

69. If \( u = \log \sin x \) then \( \frac{du}{dx} \) is equal to

(A) \( \cos x \)  \hspace{1cm} (B) \( \cos ec x \)  \hspace{1cm} (C) \( \sec x \)  \hspace{1cm} (D) \( \cot x \)

70. If \( \{a_n\}_{n=0}^\infty \) converges to \( a \) and for all \( n, a_n \geq 0 \), then \( \{\sqrt{a_n}\}_{n=0}^\infty \) is

(A) Converges \( \sqrt{a} \)  \hspace{1cm} (B) Diverges to \( \sqrt{a} \)  \hspace{1cm} (C) Converges to \( a \)  \hspace{1cm} (D) Diverges to \( a \)

71. (i) Every Convergent is a Cauchy sequence
(ii) Every Cauchy sequence is a convergent sequence
(A) (i) is true  \hspace{1cm} (B) (ii) is true
(C) (i) and (ii) both are false  \hspace{1cm} (D) Both (i) and (ii) are true

72. Find the value of \( n \) such that \( \lim_{x \to 3} \frac{x^n - 3^n}{x - 3} = 108 \)

(A) 3  \hspace{1cm} (B) 2  \hspace{1cm} (C) 4  \hspace{1cm} (D) 2

73. The \( n^{th} \) term of the sequence \( \{1, \frac{5}{2}, \frac{5}{3}, \frac{9}{4}, \frac{9}{5}, \ldots\} \) is

(A) \( \frac{2n + (-1)^n}{2n} \)  \hspace{1cm} (B) \( \frac{2n + (-1)^n}{n} \)  \hspace{1cm} (C) \( \frac{2n + (1)^n}{2n} \)  \hspace{1cm} (D) \( \frac{2n + (1)^n}{n} \)

74. A sequence \( \{a_n\} \) is bounded iff there is a real number \( S \) such that

(A) \( |a_n| \leq S \), for all \( n \)  \hspace{1cm} (B) \( |a_n| \geq S \), for all \( n \)
(C) \( |a_n| = S \), for all \( n \)  \hspace{1cm} (D) \( |a_n| < S \), for all \( n \)

75. The series \( \sum_{n=1}^{\infty} \frac{(-1)^n}{2n - 1} \) is

(A) Convergent  \hspace{1cm} (B) Divergent
(C) Unbounded  \hspace{1cm} (D) Absolutely convergent sequence
76. A square matrix 'A' is said to be skew symmetric if
   (A) \( A^T = A \)  (B) \( A^T = -A \)  (C) \( A^2 = A \)  (D) \( A^2 = -A \)

77. If \( A = \begin{bmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{bmatrix} \) then \(|A|\) is equal to
   (A) \((a - b)(b - c)(c - a)\)  (B) \((a - b)(b + c)(c - a)\)
   (C) \((a - b)(b - c)(c + a)\)  (D) \((a - b)(b - c)(c - a)(a + b + c)\)

78. If \( A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \) then the trace of \( A^{-1} \) is
   (A) 4  (B) 5  (C) 2.5  (D) 2

79. If \( A \) is a matrix of order \( 4 \times 4 \) and if it is singular, then the rank of \( A \) is
   (A) 0  (B) 4  (C) less than 4  (D) cannot be determined

80. For the following system of homogeneous equations \( AX = b \), if determinant of \( A \) is non-zero, then the system of equations possess
   (A) unique solution  (B) infinite number of solutions
   (C) no solution  (D) both (B) and (C)

81. Sum of the roots \( \alpha \) and \( \beta \) of the equation \( ax^2 + bx + c = 0 \) is
   (A) \( \frac{-b}{a} \)  (B) \( \frac{-a}{b} \)  (C) \( \frac{c}{a} \)  (D) \( \frac{-b^2}{a} \)

82. The roots of \( \alpha \) and \( \beta \) of the equation \( ax^2 + bx + c = 0 \) are imaginary if
   (A) \( b^2 - 4ac = 0 \)  (B) \( b^2 - 4ac < 0 \)  (C) \( b^2 - 4ac > 0 \)  (D) \( b^2 - 4ac \neq 0 \)

83. If discriminant \( D = b^2 - 4ac = 0 \) for \( ax^2 + bx + c = 0 \) then the roots are
   (A) real and imaginary  (B) real and equal
   (C) imaginary  (D) real
84. The set of values of \( k \) for equation \( kx^2 - 6x - 2 = 0 \) has real roots when

(A) \( k = \frac{-9}{2} \) \hspace{1cm} (B) \( k \geq \frac{-9}{2} \) \hspace{1cm} (C) \( k < \frac{-9}{2} \) \hspace{1cm} (D) \( k = \frac{7}{2} \)

85. If sum and product of roots of quadratic equation \( ax^2 - 5x + c = 0 \) are equal to 10, then the values of \( a \) and \( c \) are

(A) \( \frac{-1}{2} \) and \( \frac{1}{2} \) \hspace{1cm} (B) \( \frac{-1}{2} \) and \( \frac{-1}{2} \) \hspace{1cm} (C) \( \frac{1}{2} \) and 5 \hspace{1cm} (D) \( \frac{-1}{2} \) and \(-5\)

86. If \( \alpha \) and \( \beta \) are roots of the equation \( ax^2 + bx + c = 0 \), then the value of \( \alpha^2 + \beta^2 \) is

(A) \( \frac{b^2}{a^2} \) \hspace{1cm} (B) \( \frac{2ac}{a^2} \) \hspace{1cm} (C) \( \frac{b^2 - 2ac}{a^2} \) \hspace{1cm} (D) \( \frac{b^2 + 2ac}{a^2} \)

87. Evaluate \( \int_0^1 x(1-x)^n \ dx \)

(A) \( \frac{1}{(n-1)(n+2)} \) \hspace{1cm} (B) \( \frac{1}{(n+1)(n+2)} \) \hspace{1cm} (C) \( \frac{1}{(n+1)(n+2)} \) \hspace{1cm} (D) \( \frac{1}{(n-1)(n-2)} \)

88. Evaluate \( \int e^x + \frac{1}{e^x} \ dx \)

(A) \( xe^x + c \) \hspace{1cm} (B) \( x - e^{-x} + c \) \hspace{1cm} (C) \( xe^{-x} + c \) \hspace{1cm} (D) \( e^{-x} - x + c \)

89. Evaluate \( \int x 5^x dx \)

(A) \( \frac{x 5^x}{\log 5} - \frac{5^x}{(\log 5)^2} + c \) \hspace{1cm} (B) \( \frac{x 5^x}{\log 5} + \frac{5^x}{(\log 5)^2} + c \)

(C) \( \frac{x 6^x}{\log 6} - \frac{6^x}{(\log 6)^2} + c \) \hspace{1cm} (D) \( \frac{x 6^x}{\log 6} + \frac{6^x}{(\log 6)^2} + c \)

90. Evaluate \( \int \frac{dx}{x^2 - a^2} \)

(A) \( \frac{1}{a} \log \left( \frac{x-a}{x+a} \right) + c \) \hspace{1cm} (B) \( \frac{1}{2a} \log \left( \frac{x-a}{x+a} \right) + c \)

(C) \( \frac{1}{a} \log \left( \frac{x+a}{x-a} \right) + c \) \hspace{1cm} (D) \( \frac{1}{2a} \log \left( \frac{x+a}{x-a} \right) + c \)

91. Let \( A = \{4, 8, 12\} \) and \( B = \{1, 2, 3\} \). Define \( f : A \rightarrow B \) by \( f(x) = \frac{x}{4} \) where \( x \in A \). Then \( f \) is

(A) into function \hspace{2cm} (B) onto function

(C) one-one onto function \hspace{2cm} (D) constant function

13 \hspace{1cm} 375
92. If \( f(x) = \frac{1}{x} \), \( g(x) = x^2 + 3 \), then the composite function \( f \circ g \) is equal to

\[
\begin{align*}
(A) \quad & \frac{1}{x + 3} \\
(B) \quad & \frac{1}{x^2 + 3} \\
(C) \quad & \frac{1}{x} \\
(D) \quad & \frac{1}{x^2}
\end{align*}
\]

93. The value of \( a \) when \( 5x^3 - 2x + a \) is divided by \( (x - 2) \) is

\[
\begin{align*}
(A) \quad & -28 \\
(B) \quad & -29 \\
(C) \quad & -27 \\
(D) \quad & 27
\end{align*}
\]

94. The value of \( m \) when \( (x + 1) \) is a factor of \( x^3 + mx^2 + 19x + 12 \) is

\[
\begin{align*}
(A) \quad & 8 \\
(B) \quad & -8 \\
(C) \quad & 7 \\
(D) \quad & -7
\end{align*}
\]

95. The value of \( 12^2 + 13^2 + \ldots + 40^2 \) is

\[
\begin{align*}
(A) \quad & 21635 \\
(B) \quad & 21636 \\
(C) \quad & 21634 \\
(D) \quad & 21630
\end{align*}
\]

96. The value of \( 1 + 8 + 27 + \ldots + 8000 \) is

\[
\begin{align*}
(A) \quad & 44100 \\
(B) \quad & 43100 \\
(C) \quad & 42100 \\
(D) \quad & 40100
\end{align*}
\]

97. If \( \frac{1}{1 + \log_x 10} = \frac{3}{4} \), then the value of \( x \) is equal to

\[
\begin{align*}
(A) \quad & 100 \\
(B) \quad & 1200 \\
(C) \quad & 1400 \\
(D) \quad & 1000
\end{align*}
\]

98. The value of \( \log \frac{11}{5} + \log \frac{490}{297} - 2 \log \frac{7}{9} \) is

\[
\begin{align*}
(A) \quad & -\log 6 \\
(B) \quad & \log 7 \\
(C) \quad & \log 8 \\
(D) \quad & \log 9
\end{align*}
\]

99. The rank of the matrix

\[
\begin{bmatrix}
1 & 2 & 3 & -1 \\
2 & 4 & 6 & -2 \\
3 & 6 & 9 & -3
\end{bmatrix}
\]

is equal to

\[
\begin{align*}
(A) \quad & 3 \\
(B) \quad & 2 \\
(C) \quad & 1 \\
(D) \quad & 4
\end{align*}
\]

100. If \( z_1 = 2 + i \) and \( z_2 = 3 - 2i \) then the conjugate of \( z_1 z_2 \) is

\[
\begin{align*}
(A) \quad & 8 - i \\
(B) \quad & 8 + 2i \\
(C) \quad & 8 + i \\
(D) \quad & 8 + 3i
\end{align*}
\]